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THE LIBRARY OF THE UNIVERSITY OF CALIFORNIA LOS ANGELES

OBSERVATIONS

ON

REVERSIONARY PAYMENTS;

ON

SCHEMES for providing ANNUITIES
for WIDOWS, and for Perſons in OLD AGE;

ON

The METHOD of Calculating the VALUES
of ASSURANCES on LIVES;

AND ON

THE NATIONAL DEBT.

To which are added,

FOUR ESSAYS

On different Subjects in the Doctrine of LIFE
ANNUITIES and POLITICAL ARITHMETICK.

ALSO,

AN APPENDIX,

Containing a complete Set of TABLES; particularly,
Four New Tables, ſhewing the Probabilities of Life in
LONDON, NORWICH, and NORTHAMPTON; and the
Values of *joint* Lives.

BY RICHARD PRICE, D.D. F.R.S.

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Printed for T. CADELL, in the Strand.

INTRODUCTION.

BEFORE the Reader enters upon this Work, it will not be improper to give him the following information concerning it.

A few years ago, many gentlemen, of the firſt eminence in the law, formed themſelves into a *Society*, for providing annuities for the widows of all ſuch perſons in judicial offices, barriſters, civilians, and ſollicitors, as ſhould chuſe to become members. A plan was agreed upon and printed; but, ſome doubts happening to ariſe with reſpect to it, the directors reſolved to aſk the opinion and advice of three gentlemen, well known for their ſkill in calculation. This occaſioned a further reference to me; and the iſſue was, that the plan being found to be inſufficient, the whole deſign was laid aſide.

About the ſame time, ſeveral other ſocieties were formed with the ſame views; but all on plans alike improper and inſufficient.

Finding, therefore, that the public wanted information on this ſubject, I was led to undertake this work; imagining, that it might be ſoon finiſhed, and that all I could ſay might be brought into a very narrow compaſs. But in this I have been much miſtaken. A deſign, which I at firſt thought would give little trouble, has carried me far into a very wide field of enquiry; and engaged me in many calculations that have taken up much time and labour. I ſhall, however, be ſufficiently rewarded for my labour, ſhould it prove the means of preventing any part of that diſtreſs, which is likely to be hereafter produced by the ſocieties now ſubſiſting for the benefit of widows.——I have proved the inadequateneſs of their plans, by undeniable facts and mathematical demonſtration (*a*).—I have, further, given an account of ſome of the beſt plans, that are conſiſtent with a ſufficient probability of permanency and ſucceſs.——Should, therefore, any of theſe ſo-

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(*a*) In the note p. 68, I take notice of the five guineas *fine* required by ſome of theſe ſocieties at admiſſion; and mention ſome reaſons for not making any particular allowance for it. But I have ſince learnt, that it is indeed no more, than the firſt of the yearly payments, which I have always ſuppoſed to begin immediately. No allowance, therefore, was neceſſary to be made for it.

cieties determine to reform themſelves; or ſhould any inſtitutions of the ſame kind be hereafter eſtabliſhed, they will here find direction and aſſiſtance (*b*).

In

(*b*) I have lately learnt, that Mr. *Cadell*, the publiſher of this work, and alſo Mr. *Becket*, Bookſeller in the *Strand*, are commiſſioned to deliver in *London*, printed accounts of the ſcheme of a ſociety, eſtabliſhed five years ago at *Amſterdam*, for granting annuities on ſurvivorſhip.——I cannot ſatisfy my own mind without introducing here, though an improper place, the following remarks on this ſcheme.

From the ſolution of Queſtions I and IV. in the Firſt Chapter of the following Work, it may be gathered, that, (reckoning intereſt at 3½ *per cent.* and the probabilities of life as they are in Tables III. IV. and V. in the *Appendix*) the value of an annuity of 1*l.* for life, to be enjoyed by a perſon aged 20, provided he ſurvives another perſon aged 60, is 8*l.* 16*s.* 6*d.* in one preſent payment; and 18*s.* 6*d.* in annual payments, during the two joint lives: the firſt payment to be made immediately. A *ſingle payment*, therefore, of 130 *florins*, entitles to an annuity of 15 *florins*; and an *annual* payment of 110 *florins*, to an annuity of 119 *florins*; and both together, to an annuity of 134 *florins*. If the annual payments are to be made, not during the *joint* lives, but during the whole continuance of the oldeſt *ſingle* life, they will, together with the ſingle payment, entitle to an annuity of 144 *florins*. But this ſociety promiſes, for theſe payments, an annuity of 100 *florins*, if the oldeſt life fails in the firſt year after admiſſion; 200 *florins*, if it fails in the 2d year; 300 *florins*, if it fails in the third; 400 *florins*, if it fails in the 4th; and 500 *florins*, if it fails in the *fifth* year, or at any time afterwards. It is, therefore, evident that the ſcheme of this ſociety is, in this inſtance, groſsly defective. There are other inſtances in which it is even more defective; and the *whole* of it, like the ſchemes of moſt of the *London*

In Queſtion VI. Chap. I. a general method is deſcribed of finding the values, in *ſingle* and *annual* payments, of all life-annuities which are to begin after a given term of years; and, in the 4th Section of the 2d Chapter, the plans of the ſocieties for granting ſuch annuities are particularly conſidered, and proved to be extremely deficient.—— Indeed, the general diſpoſition which has lately ſhewn itſelf to encourage theſe ſocieties, is a matter of the moſt ſerious concern; and ought, I think, to be taken under the notice of the Legiſlature. The leading perſons among the *preſent* members, will be the *firſt*

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don ſocieties, appears to have been contrived by perſons who had no principles to go upon. And yet it has been much encouraged. Many have entered themſelves into it from different parts of *Europe*; and the printed plan acquaints us, that it is now in poſſeſſion of an annual income of 200,000 *florins*. What diſappointment then muſt it in time produce?——It is provided by its rules, that the terms of admiſſion ſhall become leſs and leſs advantageous, the longer it has ſubſiſted; juſt as if the value of the annuities it promiſes depended, not on the probabilities of life, and the improvement to be made of money, but on the age of the ſociety.——I have taken notice of a ſimilar abſurdity in the rules of our own ſocieties. But it is eaſy to ſee what is meant by it.

[*] Mr. *Cadell* can procure from his correſpondents in *Holland*, any information for thoſe who may want to know more of this ſociety. But indeed I ſhould be ſorry to find it much enquired after in LONDON.

annuitants; and they are ſure of being gainers: and the more inſufficient the ſcheme is, on which a ſociety is formed, the greater will be the gains of the firſt annuitants. The ſame principle, therefore, that has produced and kept up other *bubbles*, has a tendency to preſerve and promote theſe; and, for this reaſon, it is to be feared, that, in the preſent caſe, no arguments will be attended with any effect (*c*). The conſideration, that "the gain made by ſome in theſe ſocieties, "will be ſo much plunder taken from "others," ought immediately to engage all to withdraw from them, who have any regard to juſtice and humanity; but experience proves, that this argument, when oppoſed to

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(*c*) This apprehenſion has been verified by fact.—At the beginning of laſt winter a letter was publiſhed to the *Provident Society*, containing a clear proof of the inſufficiency of the plans of all theſe ſocieties. It was at leaſt to be expected, that ſuch a publication would prevent the riſe of new ſocieties, formed on *more* inadequate plans. But this was ſo far from being the effect, that, ſoon afterwards, a ſociety ſprung up which calls itſelf the *Rational Annuity Society*; and which, though it does not take *half* the values of the annuities it promiſes, has had the ſhameleſſneſs to *aſſure* the public, that it is formed on a plan *inconteſtably durable*. The *Conſolidated*, the *Public Annuitant*, and the *Weſtminſter Union* Societies, are yet *worſe* inſtitutions, which have been ſince formed; and there may, for ought I know, be many more: for, indeed, all LONDON ſeems to be now entering into aſſociations of this kind.

private intereſt, is apt to be too feeble in its influence.

It cannot be ſaid with preciſion, how long theſe ſocieties may continue their payments to annuitants, after beginning them. A continued increaſe, and a great proportion of young members, may ſupport them for a longer time than I can foreſee. But the longer they are ſupported by ſuch means, the more miſchief they muſt occaſion.—So, a tradeſman, who ſells cheaper than he buys, may be kept up many years by increaſing buſineſs and credit; but he will be all the while *accumulating* diſtreſs; and the longer he goes on, the more extenſive ruin he will produce at laſt.

In the latter end of the firſt chapter, I have ſtated very particularly, the method of computing the values of *aſſurances* on lives and ſurvivorſhips, in all caſes where no more than two lives are concerned: and, in the 3d Eſſay, I have pointed out a conſiderable error, into which there is danger of falling in computing ſome of theſe values. The ſocieties and offices for tranſacting buſineſs in this way, are very uſeful; and it is neceſſary that they ſhould go upon the beſt principles, and poſſeſs all the information that can be given them.

But there is no part of this work in which the public is ſo much concerned, as the 3d Chapter. It will be there proved, that had the ſums raiſed for public ſervices ſince the REVOLUTION, been much greater than they have been, the increaſe of the public debts to their preſent ſtate might have been prevented in the eaſieſt manner, and at a trifling expence. A method, likewiſe, of reducing within due bounds theſe debts, heavy as they now are, will be propoſed.——All competent judges will, I believe, ſee, that this method, being founded on the moſt perfect improvement that can be made of money, is the moſt expeditious and effectual that the natures of things admit of. Nor, in my opinion, if the nation is not yet too near the *limit* of its reſources, can there be any *good* reaſon againſt carrying it into execution.——It is well known, to what prodigious ſums, money, improved for ſome time at *compound intereſt*, will increaſe (*a*). A ſtate, if there is no miſ-

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(*a*) A *penny*, put out to 5 *per cent.* coumpound intereſt at our Saviour's birth, would, by this time, have increaſed to more money than would be contained in 150 millions of globes, each equal to the earth in magnitude, and all ſolid gold.

application of money, muſt neceſſarily make this improvement of any ſavings, which can be applied to the payment of its debts. It need never, therefore, be under any difficulties; for, with the *ſmalleſt* ſavings, it may, in as little time as its intereſt can require, pay off the *largeſt* debts.——Several of the obſervations I have made on this ſubject, have not, perhaps, been duly conſidered. Could they engage the attention of the managers of our public affairs, they might, I think, be of ſome ſervice. But this, I am ſenſible, I cannot expect. I have, however, in ſome degree, ſatisfied my own mind; and I ſhall always reflect with pleaſure, that, in this part of the following treatiſe, I have endeavoured to convey to the public, an information which is of particular importance to it.

In the *firſt* Eſſay I have made many obſerſervations on the expectations of lives, the pernicious influence of great towns on health, and manners, and population; the increaſe of mankind; and other ſubjects in the doctrine of Annuities and Political Arithmetick.——In the Laſt Eſſay I have ſtated carefully the proper method of forming tables of the probabilities of human life, from given obſervations;

tions: And, in the *Appendix*, beſides ſeveral new Tables, I have thought it neceſſary to give Mr. *Simpſon*'s Tables of the values and expectations of LONDON lives; and all the other Tables which can be wanted in the peruſal of this work.——I have alſo, in the *Appendix*, given the Demonſtrations of the Anſwers to the *Queſtions* in Chap. I. Theſe Demonſtrations I have choſen to keep out of ſight in the body of the work, in order to avoid diſcouraging ſuch readers as may be unacquainted with mathematics.

Upon the whole. A great part of this work is, I believe, new; and I am in hopes alſo, that it will be found to contain ſome improvements in thoſe branches of philoſophical enquiry, which are the ſubjects of it.

The Reader is deſired to correct the following Error.

In page 148, line 11. *inſtead of* 11, *read* 9 years purchaſe; *and in the next line*, *inſtead of* 6½, *read* 15½ years purchaſe.

CHAP. I.

Queſtions relating to Schemes for granting Reverſionary *Annuities, and the Values of Aſſurances on Lives.*

QUESTION I.

"A Set of married men enter into a "ſociety for ſecuring annuities to "their widows. What ſum of "money, in a ſingle preſent payment, ought every member to "contribute, in order to entitle his widow "to an annuity of 30*l. per annum* for her life, "eſtimating intereſt at 4 *per cent?*"

ANSWER.

It is evident, that the value of ſuch an expectation is different, according to the different ages of the purchaſers, and the proportion of the age of the wife to that of the huſband. Let us then ſuppoſe, that every perſon in ſuch a ſociety is of the ſame age with his wife, and that one with another all the members when they enter may be reckoned 40 years of age, as many entering above this age as below it. It has been demonſtrated by Mr. *De Moivre* and Mr. *Simpſon*, that "the value of an annuity on the *joint continuance* of any two lives, ſubtracted from "the value of an annuity on the life in expectation," gives the true preſent value of an annuity on what may happen to remain of the latter of the two lives after the other.

In the preſent caſe, the value of an annuity to be enjoyed during the *joint continuance* of two lives, each (*a*) 40, (*b*) is 9.826, according

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(*a*) See Table VII. Appendix.

(*b*) The values of *joint* lives and reverſions, as deduced from the *Breſlaw* obſervations, are not given in any part of this work from Mr. *De Moivre*'s rules in his treatiſe on annuities on lives. For theſe rules are approximations, which give reſults ſo far from the truth, as to be, not only uſeleſs, but dangerous. In the ſecond eſſay in the Appendix, a particular account of this will be given, and alſo of the method in which theſe values have been calculated.

Mr. *De Moivre* has calculated the values of *ſingle* lives, on the ſuppoſition of an *equal decrement of life* thro' all its ſtages till the age of 86, which he conſidered as the utmoſt probable extent of life. Thus; let there be 56 perſons alive at 30 years of age. It is ſuppoſed that one will die every year till, in 56 years, they will be all dead. The ſame will happen to 46 at 40, in 46 years. To 36 at 50, in 36 years, and ſo on for all other ages. The number of years which a given life wants of 86, he calls the *complement* of that life. Fifty-ſix, therefore, is the *complement* of 30; 46 of 40, and 36 of 50.

This hypotheſis eaſes very much the labour of calculating the values of lives; and it is ſo conformable to Dr. *Halley*'s table of obſervations, that there is little or no reaſon ing to the probabilities of life in the table of obſervations formed by Dr. *Halley*, from the bills of mortality of *Breſlaw* in *Sileſia*. The value of a ſingle life 40 years of age, as given by Mr. *De Moivre*, agreeably to the ſame table, is 13.20 (*a*); and the former ſubtracted from the latter, leaves 3.37, or the true number of years purchaſe, which ought to be paid for any given annuity, to be enjoyed by a

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ſon for diſtinguiſhing between the values of lives as deduced from this Table, and the ſame values deduced from the hypotheſis.

In order to avoid putting the reader to trouble, I have given this table at the end of this work. And I have alſo given two other tables which I have formed from the bills of mortality at *Northampton* and *Norwich*. Theſe laſt tables anſwer more nearly to Mr. *De Moivre*'s hypotheſis than even Dr. *Halley*'s table; and the difference between the values of *ſingle* and *joint* lives by the *hypotheſis*, and the ſame values computed ſtrictly from the tables, is generally leſs in theſe tables than in Dr. *Halley*'s, as will be ſhewn in the laſt Eſſay. When, therefore, in the courſe of this work the values of *ſingle* and *joint* lives are mentioned, as given agreeably to Dr. *Halley*'s table, it muſt be underſtood, that they are taken from Tables VI. and VII. in the Appendix, and given in ſtrict agreement only to the *hypotheſis*; and that for this reaſon, they are in reality ſtill more conformable to the *Northampton* and *Norwich* tables.

The inhabitants of *London*, as is well known, not living ſo long as the reſt of mankind, the values of *ſingle* and *joint* lives there, are conſiderably leſs than in any other place where obſervations have been kept. Whenever, therefore, I have had *London* lives in view, I have given particular notice of it, and taken their values from Mr. *Simpſon*, who has calculated them with much accuracy from the *London* tables of obſervation. See Tables X. and XI.

(*a*) See Table VI. Appendix.

perſon 40 years of age, *provided* he ſurvives another perſon of the ſame age, intereſt being reckoned at 4 *per cent. per annum*. The annuity, therefore, propoſed in this queſtion being 30*l.* the preſent value of it is 30 multiplied by 3.37, or 101*l.* 2*s.*

By calculating from Mr. *Simpſon*'s tables (*a*), formed from the bills of mortality of *London*, this value comes out 102*l.*

The difference in the value of the reverſion will be inconſiderable, whether the common age is taken a few years more or leſs than 40. Thus married men of 30 ought not, according to Dr. *Halley*'s table, to give two fifths of a year's purchaſe more, for any given reverſionary annuity for their wives, than married men of 50, provided they are of the ſame ages with their wives; and one quarter more, according to Mr. *Simpſon*'s table. If the wives are younger (as is generally the caſe) there will indeed be a conſiderable difference; for the value now determined would be 120*l.* according to the *Breſlaw* obſervations, ſuppoſing the two lives to be 40 and 33, or that wives are one with another ſeven years younger than their huſbands; and 118*l.* 10*s.* according to the *London* obſervations.

(*a*) See Table X. and XI. Appendix.

QUESTION II.

" Suppoſing ſuch a ſociety as that deſcrib-
" ed in the preceding queſtion, to be limit-
" ted to a certain number of members, and
" conſtantly kept up to that number, by the
" admiſſion of new members as old ones are
" loſt, in conſequence of their own deaths,
" and the deaths of their wives: What is the
" number of annuitants which, in ſome time
" after its eſtabliſhment, will come to be
" conſtantly upon it?"

ANSWER.

Since every marriage produces either a widow or widower; and ſince all marriages taken together would produce as many widows as widowers, were every man and his wife of the ſame age, and the chance equal which ſhall die firſt; it is evident, that the number of widows that have ever exiſted in the world would, in this caſe, be equal to *half* the number of marriages. And what would take place in the world muſt alſo, on the ſame ſuppoſitions, take place in this ſociety. ——In other words; every *other* perſon in ſuch a ſociety leaving a widow, there muſt ariſe from it a number of widows equal to half its own number. — But this does not determine what number, all living at one and the ſame time, the ſociety may expect will come to be conſtantly upon it. For if every widow lived no more than a year, the ſociety would never have more annuitants upon it than came on in a year. And on the contrary, if none ever died, the number of annuitants would go on increaſing for ever.—'Tis, therefore, neceſſary, in order to anſwer the preſent enquiry, to determine how long the *duration* of *ſurvivorſhip* between perſons of equal ages will be, compared with the *duration* of *marriage*. And the truth is, that, ſuppoſing the probabilities of life to decreaſe uniformly (*a*), the former is equal to the latter; and conſequently, that the number of *ſurvivors*, or (which is the ſame, ſuppoſing no ſecond marriages) of *widows* and *widowers* alive together, which will ariſe from any given ſet of ſuch marriages conſtantly kept up, will be equal to the whole number of marriages; or *half* of them (the number of widows in particular) equal to *half*

(*a*) That is, ſuppoſing that out of any given number alive at any age the ſame number will die every year 'till all are dead. See the preceding note. That on this hypotheſis, the duration of ſurvivorſhip is equal to the duration of marriage, when the ages are equal; or, in other words, that the *expectation* of two joint lives, the ages being equal, is the ſame with the *expectation* of ſurvivorſhip, may be learnt from the 18th and 20th problems of Mr. *De Moivre*'s treatiſe on annuities; and a demonſtration of it, together with a particular explanation of this ſubject, may be found at the beginning of the firſt Eſſay to which I muſt beg the reader to turn, if he is at any loſs about the full meaning of what is here ſaid.

the number of marriages.—Now, it appears that the decreaſe in the probabilities of life, is in fact nearly uniform. According to the *Breſlaw*, the *Northampton* and *Norwich* tables of obſervation, almoſt the ſame numbers die every year from 20 years of age to 77 (*a*). After this, indeed, fewer die, and the rate of decreaſe in the probabilities of life is retarded. But this deviation from the hypotheſis is inconſiderable; and its effect, in the preſent caſe, is to render the duration of ſurvivorſhip *longer* than it would otherwiſe be. According to the *London* table of obſervations, the numbers dying every year begin to grow leſs at 50 years of age; and from hence to extreme old age, there is a conſtant retardation in the decreaſe of the probabilities of life (*b*). Upon the whole, therefore, it appears in anſwer to the preſent queſtion, that "according to the *three* "*former tables* of obſervations, and ſuppo-"ſing no widows to marry, the number "enquired after is *ſomewhat greater* than "half the number of the ſociety; but, ac-"cording to the *London table*, a *good deal* "*greater*."

It muſt be carefully remembered, that this has been determined on the ſuppoſition, that

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(*a*) See Tables III. IV. and V. Appendix.

(*b*) The reaſon of this difference between the *London* and other tables, will be given at the end of the fourth Eſſay.

huſbands and their wives are of equal ages, and that in this caſe it becomes an equal chance which ſhall die firſt. In reality neither of theſe ſuppoſitions is juſt. Huſbands in general are older than their wives; and in equal ages the mortality of males has been found to be greater than the mortality of females. For both theſe reaſons, it is much more than an equal chance that the huſband will die before his wife, or that the woman ſhall be the ſurvivor of a marriage, and not the man. This will increaſe conſiderably the duration of ſurvivorſhip on the part of the woman, and conſequently the number enquired after in this queſtion. The marriage of widows will alſo diminiſh this number, and the operation of theſe cauſes will be different in different ſituations. But it is by no means to be expected (in the ſituation of the ſocieties I have in view) that the diminution from the latter cauſe will be conſiderable enough, to overbalance the operation of all the other cauſes which have been mentioned, and reduce the number under conſideration ſo low, as half the number of marriages (*a*).

SCHOLIUM.

In *London* it appears, that there is a retardation of the decreaſe in the probabilities

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(*a*) It will be obſerved hereafter, that this obſervation has been found to be true in fact.

of life, which renders the duration of ſurvivorſhip between two lives of equal ages, conſiderably longer than their joint continuance. It ſeems worth obſerving, that this is the reaſon why, though the probabilities of life, and therefore the values of ſingle and joint lives, are leſs in *London* than in other places, yet the values of reverſions depending on ſurvivorſhips, are in ſome caſes greater there. It is proper to add, that this likewiſe is the reaſon why, in calculating the values of joint lives and reverſions, the preſent value of an annuity payable yearly to the ſurvivor of two equal lives, may come out equal to, or even greater than, the preſent value of a like annuity for the joint lives. As an annuity, during ſuch ſurvivorſhip, will probably not become payable for ſome years, and therefore the money given for it will have time to accumulate, it is manifeſt, that the value of it could never be equal to the value of an annuity on the joint lives, the payment of which begins immediately, were not the obſervation now made true.

QUESTION III.

" Such a ſociety as that deſcribed in the
" preceding queſtions being ſuppoſed, in
" what time will the number of annuitants
" upon it come to a *maximum?*"

ANS-

ANSWER.

In order to be more clear in anſwering this queſtion, I will firſt ſuppoſe the ſociety to comprehend in it from its firſt eſtabliſhment, *all* the married perſons of *all* ages in any town or country, where the number of people continue conſtantly the ſame. In this caſe, the whole collective body of members will be at their greateſt age, at the time of the eſtabliſhment of the ſociety; and the number of members, together with the number of widows left every year, will, taking one year with another, admit of no increaſe or diminution. The number of widows in life together, derived from any given number coming on a ſociety every year, will increaſe continually, till as many die off as are added every year; that is, till they come to die off as faſt as poſſible. But they cannot die off as faſt as poſſible, till the whole collective body of widows are at their greateſt age; or, till there is among them the greateſt number poſſible of the oldeſt widows; and, therefore, not till there has been time for an acceſſion to the oldeſt widows, from the youngeſt part of the widows that come on annually.

Let us, for the ſake of greater preciſion, divide the whole medium of widows that come on every year, into different claſſes according to their different ages, and ſuppoſe ſome to be left at 56 years of age, ſome at 46, ſome at 36, and ſome at 26. The widows, conſtantly in life together, derived from the firſt claſs, will come to their greateſt age, and to a *maximum*, in 30 years, ſuppoſing with Mr. *De Moivre*, 86 to be the utmoſt extent of life. The ſame will happen to the ſecond claſs in 40 years, and to the third in 50 years (*a*). But the whole body, compoſed of theſe claſſes, will not come to a *maximum*, till the ſame happens to the fourth or youngeſt claſs; that is, not till the end of 60 years. After this, the affairs of the ſociety will become *ſtationary*, and the number of annuitants upon it of all ages will keep always nearly the ſame.

Such is the anſwer to this queſtion, ſuppoſing a ſociety to begin with its complete number of members, conſiſting of married perſons of all ages, in the ſame proportions to one another, with the proportions in which they exiſt in the world.——If it begins with its complete number of members, but at the ſame time admits none above a particular age: If, for inſtance, it begins with 200 members all under 50, and afterwards limits itſelf to this number, and keeps it up by admitting every year, at all ages between 26 and 50, new members as old ones drop off;

(*a*) In the *Appendix*, note (*a*), a rule is given, by which the numbers alive at the end of any particular number of years may be very eaſily determined.

in this caſe, the period neceſſary to bring on the *maximum* of annuitants will be juſt doubled. For, in the firſt place, the whole collective body of members will be 60 years in getting to their greateſt age, as may eaſily appear from what has been juſt ſaid. The annual medium of widows, therefore, that will come on the ſociety will increaſe continually for 60 years, it being evident, that the older any ſet of married men are, taken one with another, the faſter they will leave widows. And after this annual medium is increaſed to a *maximum*, 60 years more will be neceſſary to bring to a *maximum* the number in life together, derived from ſuch a *fixed* annual medium conſtantly coming on.——If ſuch a ſociety is any number of years in gaining its *maximum* of members, the time neceſſary to bring on the *maximum* of annuitants will be ſtill further prolonged, and will be equal to twice 60 years with that number of years added.—Moſt of the ſocieties for granting annuities to widows are of this kind; and, therefore, ſuppoſing them to gain their complete number of members in ten years, and for ever afterwards to preſerve it, the number of annuitants upon them will go on increaſing for 130 years.—It is proper, however, to be remembered, that the increaſe will be quicker at firſt, and afterwards ſlower; and that, within 20 or 30 years of the end of this term, it will be ſo ſlow as ſcarcely to be ſenſible, though ſtill real.

All who will beſtow due attention on this ſubject muſt ſee theſe deciſions to be juſt; and a demonſtration of them might be given, in a form more ſtrictly mathematical, were it neceſſary.

QUESTION IV.

"Suppoſe the members of ſuch a ſociety "as that deſcribed in the preceding queſtions, to chuſe making *annual payments during* "*the continuance of marriage*, in lieu of the "ſum which the reverſionary annuity for "their widows is worth in *preſent money*: "What ought theſe *annual payments* to be, "eſtimating intereſt at 4 *per cent*?"

ANSWER.

This will be eaſily determined, by finding what annual payments, during two joint lives of given ages, are equivalent to the value of the reverſionary annuity in *preſent money*.— Suppoſe, as in Queſtion I. the two joint lives to be each 40, and the reverſionary annuity 30*l*. *per annum*. An annual payment during the continuance of two ſuch lives is worth, according to Dr. *Halley*'s table of obſervations, 9.82 (*a*) years purchaſe. The annual

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(*a*) See Table VII.

payment then ought to be ſuch as being multiplied by 9.82, will produce (*a*) *l.* 101.1, the preſent value of the annuity in one payment by Queſtion I. Divide then *l.* 101.1 by 9.82, and the *quotient*, or *l.* 10.3 will be the anſwer.——This is very nearly the annual payment of all the members at an average, ſuppoſing equal numbers to offer themſelves for admiſſion of every age between 30 and 50. As much as ſome give leſs, others ought to give more, according to their exceſs of age. Thus, the annual payment of a married perſon, 30 years of age, ought to be *l.* 9.39; and of a perſon 50 years of age *l.* 11.33.——If the values of joint lives and the reverſionary annuity are taken agreeably to the *London* table of obſervations, theſe annual payments will be, for 30 years of age (*b*), *l.* 10.9,—for 40, *l.* 12.5,—for 50, *l.* 14.5.

If

(*a*) Particular notice ſhould be taken of the method of notation here uſed, becauſe it will be carried through the whole of this work.——The figures on the right hand of the full-point, ſignify the decimal parts of 1 *l.* Thus; *l.* 101.1, is *l.* 101 and the 10th of 1 *l.* or *l.* 101 and 2 *s.*——*l.* 9.39, is *l.* 9, and 39 hundredths of 1 *l.* or *l.* 9 : 7 *s.* : 10 *d.*——*l.* 11.33, is *l.* 11, and 33 hundredths of 1 *l.* or *l.* 11 : 6 *s.* : 7 *d.*—— In general; it ſhould be remembered, that 2 ſhillings allowed for every unit in the firſt place of decimals, and two-pence half-penny for every unit in the ſecond place of decimals, will give, nearly enough, the value of the decimal part of every ſuch expreſſion.

(*b*) The value of two joint lives of 30, taken from Table XI. is 9.6. This ſubtracted from the value of the life in expectation, or from 13.1, by Table X. gives 3.5, the

If either the rate of intereſt is ſuppoſed lower, or wives are ſuppoſed younger than their huſbands, the annual payments will be increaſed. But there is no occaſion for pointing out particularly the difference. It may be eaſily found in any caſes by the directions now given. There is, however, one obſervation which ought to be here carefully attended to.—This method of calculation ſuppoſes, that the firſt annual payment is not to be made 'till the end of a year. If it is to be made *immediately*, the value of the joint lives will be increaſed one year's purchaſe; and, therefore, in order to find in this caſe the annual payments required, the value in preſent money found by Queſt. I. muſt be divided by the value of the joint lives increaſed by unity, and, in this way, the preceding values at 4 *per cent.* according to the *Breſlaw* obſervations, will be found to be *l.* 8.62—*l.* 9.35—*l.* 10.07.—According to the *London* obſervations, *l.* 10,—*l.* 11.2,—*l.* 12.7.

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the number of years purchaſe which an annuity for a life of 30 years of age, *after* another life of the ſame age, is worth. This remainder, multiplied by 30, gives 105*l.* the value in a ſingle payment, ſuppoſing the reverſionary annuity to be 30*l.* And 105*l.* divided by 9.6, gives *l.* 10.9, the value of the ſame annuity in annual payments, during the joint continuance of the two lives, according to the *London* obſervations.—By ſimilar operations all the other values above given have been found.

QUESTION V.

"A ſociety may chuſe to make abate-
"ments in theſe annual payments, and to re-
"quire the remainder of the value of the
"reverſionary annuity to be given, in fines
"or premiums at the time of admiſſion; it
"may, for inſtance, chuſe to fix the annual
"payments of all the members to 5 guineas.
"What, in this caſe, would be the premium
"due at admiſſion, the annuity being ſup-
"poſed 30*l.* *per annum*, and intereſt being
"at 4 *per cent?*"

ANSWER.

From the whole preſent value of the annuity in one payment, ſubtract the value of 5 guineas *per annum*, during the joint lives; and the remainder will be the anſwer.

Suppoſing the joint lives both 40, the whole preſent value of the annuity in one payment is, according to the *Breſlaw* obſervations, *l.* 101.1, by Queſt. I.—The value of 5 guineas *per annum*, or of *l.* 5.25 *per annum*, during two ſuch joint lives, is 5.25, multiplied by the value of the joint lives; that is, 5.25, multiplied by 9.82, or *l.* 51.55; and this ſubtracted from *l.* 101.1, gives *l.* 49.5, the anſwer required for two lives at the age of 40.—The anſwer found in the ſame way for two lives whoſe common age is 30, is *l.* 46.5,—and for two lives at 50, 50*l.*

According to the *London* obſervations, theſe values are, for two lives at 30, *l.*54.6.—At 40, *l.*59. 4.—At 50, *l.*63. 3.

If the firſt of the annual payments is to be made immediately, the true anſwer will, in every inſtance, be the values found in the manner now directed, diminiſhed by the annual payment; or, in the preſent caſe, 5 guineas leſs than the values ſpecified.

The values, in *premiums* and *annual payments*, of any other reverſionary annuity, will be as much greater or leſs than theſe, as the annuity itſelf is greater or leſs.

QUESTION VI.

" A perſon 35 years of age wants to buy " an annuity, for what may happen to re" main of his life after 50 years of age. " What is the value of ſuch an annuity in " *ready-money*, and alſo in *annual payments*, " till he attains to the ſaid age; that is, in " annual payments for 15 years, ſubject in " the mean time to failure, ſhould his life " fail?"

ANSWER.

The preſent value of ſuch an annuity is the *preſent* value of a life at 50, in money to be received 15 years hence, and the payment of which depends on the contingency of the continuance of the given life 15 years. That is; it is equal to the value of a life at 50, multiplied by the preſent value of 1 *l.* to be received at the end of 15 years, and alſo by the probability that the given life will continue ſo long.—A life at 50, according to Mr. *De Moivre*'s valuation of lives, and reckoning intereſt at 4 *per cent.* is worth 11, 34 year's purchaſe. The preſent value of 1 *l.* to be received at the end of 15 years, is, by Table I, 0.5553. And the probability that a life at 35, will continue 15 years, is, according to the *Breſlaw* obſervations $\frac{346}{490}$ (*a*). And theſe three values, multiplied by one another, give *l.* 4.44, or the number of years purchaſe that ought to be given for the annuity.—The annuity then being ſuppoſed 50*l.* its value in preſent money is 222*l.*

(*a*) The probability that a given life ſhall continue any number of years, or attain to a *given age*, is (as is well known) the fraction, whoſe *numerator* is the number of the living in any table of obſervations oppoſite to the *given age*, and *denominator*, the number oppoſite to the preſent age of the given life.—Thus, in the preſent inſtance; 346 is the number in Dr. *Halley*'s table oppoſite to 50, and 490 the number oppoſite to 35.—$\frac{346}{490}$, (or the odds of 17 to 7) is, therefore, the probability that a perſon whoſe age is 35 ſhall attain to 50, or live 15 years. In the ſame manner it will appear, that, according to the ſame table, the probability that a perſon at this age ſhall live 25 years, is $\frac{242}{490}$; or nearly an even chance.

At *Northampton* and *Norwich* a perſon at the ſame age, has an even chance of living 26 years; but in *London*, ſcarcely 20 years. See Tables III, IV, V, and VIII. Appendix. I will add, though foreign to my preſent purpoſe, that a perſon at the ſame age has in theſe towns a better chance of living one year, than in *London*, in the proportion of 3 to 2.

In order to find this value in *annual payments,* while the given life is attaining to 50, it is neceſſary to find the value of an annuity for 15 years, ſubject to failure on the extinction of the given life. And the value of ſuch an annuity is, evidently, the laſt value ſubtracted from the value of the given life; or, in the preſent inſtance, *l.* 4.44, ſubtracted from *l.* 13.97. (See Table VI, Appendix) that is, *l.* 9.53,—222*l.* then, being the preſent value of an annuity of 50*l.* for the remainder of a life now 35, after attaining to 50; and 9.53 being the number of years purchaſe, which ought to be given for an annual payment to laſt 15 years, if a life now 35 laſts ſo long, it follows, that the value of the ſame annuity in annual payments till this life attains to 50, is 222*l.* divided by 9.53; or *l.* 23.3.

This calculation ſuppoſes, that the firſt of the annual payments is not to be made till the end of a year. If the firſt payment is made immediately, the value will be, the *ſingle payment* divided by the value of the life for the given term increaſed by unity; that is, in the preſent caſe, 222*l.* divided by 10.53; or *l.* 21.08.

If the value of the annuity is required in a ſingle payment, over and above any given annual payment; deduct the value of the annual payment from the whole value in a ſingle preſent payment, and the remainder will be the anſwer.—Thus; let 5 guineas, in the preſent inſtance, be the given annual payment for the aſſigned term; and let the enquiry be, how much more in preſent money the ſuppoſed annuity is worth. By what has been juſt ſaid, 9.53, multiplied by 5 guineas, that is, 50*l.* is the value of the annual payment; and this ſum deducted from 222*l.* leaves 172*l.* the anſwer.

If the annual payment begins immediately, its value is 10.53, multiplied by 5 guineas, and the anſwer comes out *l.* 166.75.

In this way may be found the value, in ſingle and annual payments, of any other annuity, payable to an aſſigned life, after a given term of years, taking any valuation of lives or intereſt of money. But care muſt be taken to remember, that it is the title to the annuity that will commence at the end of the given term, and that the firſt payment is not to be made 'till a year afterwards; that is, in the caſe here ſpecified, not 'till the end of 16 years.

SCHOLIUM.

The value of the *remainder* of two joint lives, after a given term of years, is likewiſe the value of 1 *l.* due at the end of the given term, multiplied by the value of two joint lives, each older by the given term than the given lives; and this product, multiplied by the probability, that the given joint lives ſhall not fail in the given term; or (which is the ſame) by the product of the two probabilities, that the ſingle lives ſhall each continue the given term. And the value of an annuity, on any given joint lives for a term of years beginning now, is this laſt value, ſubtracted from the whole preſent value of the joint lives. Thus; the value of two joint lives, one 40 years of age, and the other 50, (ſee Table VII.) is 8.91; which, multiplied by 0.6755, the value of 1 *l.* due 10 years hence, and by $\frac{445}{531}$, (the probability that a life at 30 ſhall continue 10 years) and alſo by $\frac{346}{445}$, (the probability that a life at 40 ſhall continue 10 years) gives 3.92, the preſent value of the remainder of two joint lives, aged 30 and 40, after 10 years; and this value, ſubtracted from 10.43 (the value in Table VII. of two joint lives, aged 30 and 40) leaves 6.51, their value for 10 years.

As the value of the longeſt of two lives is always the value of the *joint* lives, ſubtracted from the ſum of the values of the two *ſingle* lives; their value alſo for any *given term*, is the value of the *joint* lives for the given term, ſubtracted from the ſum of the values of the *ſingle* lives for the given term.

The truth of theſe rules may eaſily appear without particular proof. I have, however, pointed out the method of demonſtrating them in a note (*a*) at the end of this work.

(*a*) See note (B) in the Appendix.

By ſimilar operations, may be found the values of 3 or more *joint* lives, or the longeſt of *three* or more lives, for a given term of years, or of what ſhall remain of them after a given term of years.

QUESTION VII.

" The preſent value is required of an an-
" nuity to be enjoyed by one life, for what
" may happen to remain of it beyond ano-
" ther life, after a given term; that is, pro-
" vided *both* lives continue, from the pre-
" ſent time, to the end of a given term of
" years?"

ANSWER.

Find the value of the annuity for two lives greater, by the given term of years, than the given lives. Diſcount this value for the given term; and then, multiply by the probability, that the two given lives ſhall *both* continue the given term; and the product will be the anſwer.

EXAMPLE.

Let the two lives be each 30. The term ſeven years. The annuity, 10*l.* Intereſt, 4 *per cent.*——The given lives, increaſed by 7 years, become each 37. The value of two joint lives each 37, is (by Table VII) 10.25.

The value of a ſingle life at 37, is (by Table VI) 13.67. The former, ſubtracted from the latter, is 3.42, or the value of an annuity for the life of a perſon 37 years of age, after another of the ſame age, by Queſt. I.—3. 42 diſcounted for 7 years, (that is, multiplied by 0.76, the value of 1 *l.* due at the end of ſeven years, by Table I.) is 2.6.—The probability that a ſingle life at 30 ſhall continue 7 years, is (by the hypotheſis explained page 2.) $\frac{49}{56}$ (*a*). The probability, therefore, that two ſuch

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(*a*) In this caſe, it is on ſome accounts beſt, as well as eaſieſt, to take the probabilities of life from the hypotheſis, rather than immediately from the Tables.—Fifty-ſix perſons being ſuppoſed alive at 30, one will die every year, according to the hypotheſis. At the end of ſeven years then, the number of the living will be 49, and $\frac{49}{56}$, or the odds of 7 to 1, is, by note p. 18. the probability, that a life, aged 30, will continue 7 years; and this fraction, multiplied by itſelf, is the probability, that two lives of this age, ſhall *both* continue 7 years. In general, it muſt be remembered, that the probability, that any two or more events ſhall *all* happen, is the product ariſing from multiplying by one another, the probabilities of all the events taken ſeparately. The probability, therefore, that any number of perſons will *all* live any given time, is rightly found by multiplying into one another the probabilities that each of them will live that time.—It may further be of uſe to ſome, that I ſhould obſerve here, that the difference between unity and the fraction expreſſing the probability, that an event will happen, gives the probability that it will *not* happen. Thus; the probability, that a perſon 40 years of age will live 11 years, is by the *Breſlaw* Table $\frac{335}{445}$. The probability, therefore, that he will *not* live 11 years, is $\frac{335}{445}$, ſubtracted from ſuch lives ſhall both continue 7 years, is $\frac{2401}{3136}$, or, in decimals 0.765. And 2.6 multiplied by 0.765, is 1.989, the number of years purchaſe which ought to be given for an annuity, to be enjoyed by a life now 30 years of age, after a life of the ſame age, provided both continue 7 years. The annuity then being 10*l.* its preſent value is *l.* 19.89.

By ſimilar operations, it may be found, that ſuppoſing the term one year, and the ages and the rate of intereſt the ſame, the preſent value of the ſame reverſionary annuity is *l.* 32.4; and that if the term is 15 years, the value is *l.* 9.7.

For two lives each 40, theſe values are *l.* 30.33.—*l.* 17.44.—*l.* 7.3. the term being 1, 7, or 15 years.

For two lives each 50, the ſame values for the ſame terms, are *l.* 28.2,—*l.* 13.86,—*l.* 4.34 (*a*).

Theſe values, according to the *London* obſervations and Mr. *Simpſon*'s Tables of the values of ſingle and joint lives, are,

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from unity or $\frac{110}{445}$.—In like manner: The probability that two perſons aged 30, ſhall *both* live 7 years, being 0.765, the probability that they will *not* both live ſo long, or that *one or other* uf them will die in 7 years, is 0.765 ſubtracted from unity, or .235.

If any reader is unwilling to take theſe aſſertions for granted, he ſhould conſult the beginning of Mr. *De Moivre*'s, or Mr. *Simpſon*'s Treatiſes on the Doctrine of Chances, where he will find them demonſtrated.

(*a*) See Note (C) Appendix.

For 2 lives at 30—*l.*32.05—*l.*18.62—*l.*7.66.
at 40—*l.*30.7 —*l.*15.6 —*l.*5.45.
at 50—*l.*29.36—*l.*12.33—*l.*3.24.

QUESTION VIII.

“ Let the ſcheme of a ſociety for granting annuities to widows, be, that, if a member lives *a year* after admiſſion, his widow ſhall be entitled to a life annuity of 20*l.* If *ſeven* years, to 10*l.* more, or 30*l.* in the whole. If *fifteen* years; to another additional 10*l.* or 40*l.* in the whole. What ought to be the annual payments of the members for the ages of 30, 40, and 50, ſuppoſing them of the ſame ages with their wives, and allowing compound intereſt at 4 *per cent?*”

ANSWER.

According to the *hypotheſis*, explained p. 2; and, therefore, very nearly, according to the Tables of obſervation for *Breſlaw, Norwich*, and *Northampton*,

l. 8.44—*l.* 8.69—*l.* 9.05.

According to the *London* obſervations,

l. 9.41—*l.* 10.17—*l.* 10.92.

Theſe values are eaſily deduced from the values in the laſt queſtion. For example. The value of 10*l.* *per annum* for life to 40 after 40, provided the joint lives do not fail in *one* year, is, according to the *hypotheſis*, *l.* 30.33. The value of 20*l.* *per annum*, in the ſame circumſtances, is, therefore, *l.* 60.66.—In like manner, the value of 10*l.* after *ſeven* years, is *l.* 17.44. And of 10*l.* after 15 years *l.* 7.3.—Theſe values together make *l.* 85.4, or the value of the expectation, deſcribed in this queſtion, in a *ſingle preſent payment*; which, divided by 9.82, (the value by Table VII. of two joint lives at 40) gives *l.* 8.69, the value of the ſame expectation in *annual payments*, during the joint lives.—In the ſame manner may be found the anſwer in all caſes to any queſtions of this kind.

Theſe calculations ſuppoſe, that the annual payments do not begin till the end of a year. If they are to begin *immediately*, the true *annual payments* will be, as was before obſerved, the *ſingle* payments, divided by the value of the joint lives increaſed by unity; and in the preſent caſe they will be, by the *hypotheſis*,

l. 7.75—*l.* 7.9—*l.* 8.07.

By the *London* obſervations,

l. 8.52—*l.* 9.06—*l.* 9.51.

By the method of calculation now explained, may be eaſily found in all caſes, ſuppoſing the annual payments previouſly ſettled, what the reverſionary annuities are correſponding to them in value.—Thus, the annuities being the ſame with thoſe mentioned in this queſtion, the *mean* annual payments for all ages between 30 and 50, are nearly 8*l.* according to the *higheſt* probabilities of life; 9*l.* according to the *loweſt*; and 8 guineas the *medium* (*a*); intereſt being at 4 *per cent.* and the firſt payment to be made immediately.

If the mean annual payments, beginning immediately, are fixed to five guineas, the correſponding life annuities will be nearly (by the *hypotheſis*) 12*l.* if the contributor lives a year, and 24*l.* if he lives ſeven years; or (by the *London* obſervations) 12*l.* if he lives a year, and 20*l.* if he lives ſeven years (*b*).

It

(*a*) The value of this expectation, ſuppoſing married men 40 years of age, and their wives 30, is, in a *ſingle* payment, 113*l.* In annual payments beginning immediately *l.* 9.88, by the *hypotheſis*. And 107*l.*—and *l.* 10.93, by the *London* obſervations.

(*b*) If the annuities in expectation are 14*l.* provided a member lives a year, and 20*l.* provided he lives ſeven years, the proper *mean ſingle* payments for all ages, taken one with another, under 50 or 52, is 50 guineas nearly, according to all the Tables of obſervation, ſuppoſing equality of age between men and their wives. And the addition which ought to be made, on account of exceſs of age on the man's ſide is, taking the neareſt and the eaſieſt

It is obſervable, that the difference in the values of the annuities, ariſing from difference of ages, and the difference in the probabilities of life, is leſs in this queſtion than in queſtion 4th; and that, conſequently, the plan propoſed in it, is the ſafeſt, as well as the moſt equitable and encouraging, that a ſociety can adopt.

It is neceſſary to remark here further, that *yearly* payments which begin immediately, are more advantageous than *half-yearly* payments which begin immediately. Mr. *Simpſon* (in his Treatiſe on *The Doctrine of Annuities and Reverſions*, pag. 78, and alſo *in his Select Exerciſes*, p. 283) has ſhewn, that, in the caſe of life annuities, *half-yearly* payments, which begin at the end of half a year, are ¼ of a year's purchaſe better than *yearly* payments, which begin at the end of a year. And it is manifeſt, that *half-yearly* payments, which begin immediately, are no

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eaſieſt round ſums, about a guinea and ½ for every year as far as 17 years; or, in the annual payments, (ſuppoſed 5 guineas) ½ a guinea *per annum* for five years exceſs, and ½ a guinea more for every four years exceſs beyond five years, till the exceſs comes to be 17 years. And, I believe, that 60 guineas in *ſingle payments*, and ſix guineas in *annual payments* beginning immediately, may very well be ſtated as the *loweſt common* payments proper to be required, ſuppoſing all married men under 52, taken into a ſociety, without enquiring into the difference of age between them and their wives, the annuities being all along ſuppoſed to be *life* annuities, and intereſt reckoned at 4 *per cent.*

more

more than half a year's purchaſe better than thoſe which begin at the end of half a year. But *yearly* payments, which begin immediately, are a *whole year's* purchaſe better than the ſame payments to begin at the end of a year. The difference of value, therefore, between *yearly* and *half-yearly* payments, ſuppoſing both to begin immediately, is a quarter of a year's purchaſe in favour of the former.

QUESTION IX.

"The value is required of an annuity to "be enjoyed for what may happen to re- "main of one life after another, provided "the life in expectation continues a given "time?"

ANSWER.

Find by Queſtion VI. the preſent value of the annuity for the remainder of the life in expectation, after the given time, and multiply this value by the probability, that the other life ſhall fail within that time. Find alſo, by Queſtion VII. the value of the reverſion, provided *both* lives continue the given time. Add theſe values to one another, and the *ſum* will be the anſwer in a ſingle preſent payment.

EXAMPLE.

An annuity of 10*l.* for the life of a perſon now 30, is to commence at the end of 11 years (*a*), if another perſon now 40, ſhould be then dead; or, if this ſhould not happen, at the end of any year beyond 11 years in which the former ſhall happen to ſurvive the latter. What is the preſent value of ſuch an annuity, reckoning intereſt at 4 *per cent.* and taking the probabilities of life as they are in Dr. *Halley*'s Table?

The value of 10*l.* *per annum*, for the remainder of the life of a perſon now 30, after 11 years, found by Queſt. VI. is *l.* 69.43.— The probability that a perſon 40 years of age ſhall live 11 years, is, by Dr. *Halley*'s Table, $\frac{335}{445}$. The probability, therefore, that he will die in 11 years, is $\frac{335}{445}$ ſubtracted from unity (*b*), or $\frac{110}{445}$; which multiplied by *l.*69.43, gives *l.* 17.16.—The value of the reverſion, provided *both* live 11 years, found by Queſt. VII. is 17*l.* And this value added to the

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(*a*) That is, the title to the annuity is to commence at the end of 11 years, and the firſt payment to be made a year afterwards, in caſe the life in expectation ſhould continue ſo long, and the other fail. But if *both* lives ſhould continue the given term, the firſt payment is always to be made at the end of the year, in which the former life ſhall happen to ſurvive the latter. See Queſt. VI.

(*b*) See the Note, p. 23.

former, makes *l.* 34.16, the value required in a *ſingle preſent payment*; which payment divided by *l.*11.43, (the value by Table VII. of two joint lives, aged 30 and 40, with unity added) gives 3*l*; (*a*) or the value required in annual payments during the joint lives, the firſt payment to be made immediately.——If, every thing elſe being the ſame, the aſſigned term is 15 years, the value required will be 29*l.* in a *ſingle payment*, and *l.* 2.55 in *annual payments*.

QUESTION X.

"What money in hand, and alſo in an-
"nual payments during life, ought a perſon
"of an aſſigned age to give for a ſum of mo-
"ney, payable at his death to his heirs (*b*)?—
"In other words, what money in hand, and
"in annual payments during life, ought a
"perſon of a given age to pay for an *aſſu-*
"*rance* of any given ſum on his life?"

ANSWER.

Subtract the value of the life from the *perpetuity*. Multiply the remainder by the product

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(*a*) See the demonſtration of this rule in Note (D) Appendix.

(*b*) This queſtion is the ſame with Problem 16th, in Mr. *De Moivre*'s Treatiſe on Annuities, and Problem 26th, in Mr. *Simpſon*'s Select Exerciſes; but the anſwers there given are right only when applied to reverſionary *eſtates*, and therefore muſt be materially wrong, when applied to reverſionary *ſums*, as will appear from the *Scholium* to this Queſtion, and from note (E) in the Appendix.

product of the given ſum into the intereſt of 100*l.* for a year: and this laſt product, divided by 100*l.* increaſed by its intereſt for a year, will give the anſwer in a *ſingle preſent* payment. And this payment, divided by the value of the life, will give the anſwer in *annual* payments, during the continuance of the life.

Example. Let the life be 30. The ſum 100*l.* The rate of intereſt 4 *per cent*. And the valuation of lives, that in Table VI. The perpetuity, therefore (*a*), is 25. The intereſt of 100*l.* for a year, is 4*l.* 100*l.* increaſed by its intereſt for a year is 104*l.* And the value of the life 14.68.—The value of the life, ſubtracted from the perpetuity, gives 10.32, which, multiplied by the product of 100*l.* into 4, or by 400, gives 4128. And this, divided by 104, gives *l.* 39.7, the value of 100*l.* payable at the death of a perſon aged 30, in a ſingle preſent payment.—And this payment, divided by 14,68, is *l.* 2.7, the ſame value in annual payments during the continuance of the life.

Theſe values found in the ſame way, agreeably to the valuation of lives for *London*, in Table X, are *l.* 45.76, and *l.* 3.49.—If the life is 36, and intereſt 4 *per cent*. theſe values are 43*l.* and *l.* 3.1, by Table VI, and *l.* 49.6,

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(*a*) That is; the value of the *fee ſimple* of an eſtate found by dividing 100*l.* by the rate of intereſt.

and *l.* 4.1, by Table X.—If intereſt is reckoned at 3 *per cent.* the ſame values are, by Table VI, for 30 years of age, *l.* 48.14.—2.86.—For 36 years of age, *l.* 51.43, and *l.* 3.28.

It appears here, that difference of intereſt makes no conſiderable difference in the anſwers to queſtions of this kind, except when the values are required in a ſingle payment.

If the firſt of the annual payments is to be made immediately, the ſingle payment is to be divided by the value of the life, with unity added to it, agreeably to what has been already obſerved; and the annual payments in this caſe (intereſt ſuppoſed at 4 *per cent.*) will be by Table VI, for a life at 30, *l.* 2.53—At 36, *l.* 2.9.

If the payments are half yearly payments beginning immediately, the ſingle payment muſt be divided by the value of the life increaſed by $\frac{3}{4}$, or .75, (ſee Queſt. VIII.) And the half yearly payments, for the age of 36, will be half 2.9, or 1.45. And half 1.45, or .725, is likewiſe nearly the proper quarterly payments.

Again; if an annual payment, beginning immediately, of *l.* 2.9, ought (reckoning intereſt at 4 *per cent.*) to purchaſe 100 *l.* payable at the failure of a life now 36; 5 *l.* by the rule of proportion, ought to purchaſe 172 *l.* And in like manner, it may be found, that the ſame annual contribution, in half-yearly or quarterly payments, beginning immediately, ought to purchaſe 170*l.* — Theſe ſums, according to the *London* obſervations, are 132*l.* and 130*l.* nearly.

The reaſon of mentioning theſe particulars will be ſeen in the next chapter.

SCHOLIUM.

If the reverſion is not a *ſum*, but an annuity for ever, or an *eſtate* in *fee ſimple*, to be entered upon after a given life, its preſent value, *in a ſingle payment*, will be " the value " of the life ſubtracted from the perpetuity, " and the remainder multiplied by the an" nuity, or the annual rent of the eſtate."— And the value, in *annual payments*, will be, as before, the ſingle payment divided by the value of the life. — Univerſally. It ought to be remembered, that a reverſionary *eſtate*, after any given life or lives, is worth as much more than a correſponding reverſionary *ſum*, as 100*l.* increaſed by its intereſt for a year, is greater than 100*l.*—Thus, the preſent values, in ſingle and annual payments, of 4*l.* *per annum* for ever, and of 100*l.* in money after any aſſigned life, are to one another, (intereſt being at 4 *per cent.*) as 104 to 100, or 1.04 to 1.—The reaſon of this difference is, that the calculations ſuppoſe, that the reverſionary *ſum*, and the firſt yearly rent of the *eſtate*, or firſt payment of the annuity, are to be received at the ſame time, after the extinction of the lives in poſſeſſion. It is eaſy to ſee, that this is a circumſtance which muſt make the latter of moſt value. But to prevent any doubts about it, I ſhall explain it more particularly in a note in the Appendix (*a*).

QUESTION XI.

"A perſon of a given age, having an yearly income which will fail with his life, "wants to make proviſion for another perſon of a given age, in caſe the latter ſhould "happen to ſurvive. What ought the former to give in a ſingle payment, and alſo "in annual payments during their joint lives, "for a given ſum, payable at his death to "the latter?"

It is manifeſt, that the value of the given ſum in this caſe, muſt be leſs than in the caſe ſtated in the laſt Queſtion; becauſe, here the payment of it is ſuſpended on the contingency, that one life ſhall ſurvive another, whereas in the other caſe, it is *certainly* to be paid at the failure of a given life.

ANSWER.

Find, by the ſolution of problem 32d, p. 297, Mr. *Simpſon*'s Select Exerciſes, the

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(*a*) Vid. Appendix note (E).

value of an eſtate, correſponding to the given ſum, and depending on the given ſurvivorſhip. Divide this value by 1 *l.* increaſed by its intereſt for a year, and the quotient will be the value of the given ſum in a ſingle preſent payment. And the ſingle payment, divided by the value of the given joint lives, will be the anſwer in annual payments during the joint lives.

The ſolution I have referred to is as follows.

" Find the value of an annuity on two e-
" qual joint lives, whereof the common age
" is equal to the age of the older of the two
" propoſed lives; which value, ſubtract from
" the perpetuity, and take half the remain-
" der. Then ſay, as the *expectation* of the
" duration of the younger of the two lives is
" to that of the elder, ſo is the ſaid half re-
" mainder to a 4th proportional, which will
" be the number of years purchaſe to be gi-
" ven for the eſtate when the life in expec-
" tation is the oldeſt of the two. But if this
" life is the youngeſt, then add the number
" of years purchaſe juſt found to the value
" of the joint lives, and let the ſum be ſub-
" tracted from the perpetuity, and you will
" alſo have the anſwer in this caſe (*a*)".

Let

(*a*) Mr. *Simpſon* has given the following examples of this ſolution, adapted to *London* lives.——Example I. " Suppoſe the age of the *expectant* to be 40; of the poſ-" ſeſſor 30. The rate of intereſt 4 *per cent.* and the " given

Let the life in expectation be 30; and the other life 40: The ſum, 100*l.* Intereſt, 4 *per cent.* The valuation of lives, that in Table VI.

The *expectation* of the firſt life, is 28; of the ſecond life 23, by Mr. *De Moivre's hypotheſis*. The value of the joint lives is 10.43,

" given legacy 5000*l.* or 200*l.* *per annum.* Then the " value of two equal joint lives of 40, being 8.1, by " Table XI, and the perpetuity 25, the remainder or " difference will be here 16.9; whereof the half is 8.45. " Therefore, it will be as 23.6 to 19.6, ſo 8.45 to 7.02 " years purchaſe, or *l.* 1404, the required value."

Example II. " Let the age of the *expectant* be 30, of " the *poſſeſſor* 40, and the reſt as in the preceding exam- " ple. Here the value of the joint lives 30 and 40, will " be 8.8; which added to 7.02, (found above) the ſum " will be 15.82; whence the anſwer, in this caſe, is " 9.18 years purchaſe, or 1836."

I have ſhewn, that the values of reverſionary *eſtates*, and reverſionary *ſums*, are not the ſame as is here ſuppoſed.—The rule gives the true value when applied to the former; but, when applied to the latter, the values given by it muſt be divided by 1 *l.* increaſed by its intereſt for a year, as above directed.—The ſame obſervation is to be applied to Mr. *Simpſon*'s next Problem, or the 33d.

In theſe Examples, 23.6 and 19.6, are the expectations, in Table IX, of 30 and 40, according to the *London* Tables of Obſervation; and the method of finding them for any age, and from any Tables of obſervation, is explained at the beginning of the firſt Eſſay.

In Mr. *De Moivre's hypotheſis*, the expectation of a life, is always *half* the complement. See note p. 2.—Sometimes the *complement* of a life is mentioned without any view to Mr. *De Moivre*'s hypotheſis, and it then means double the *expectation* of the life whatever that may be according to any Table of obſervations.

by Table VII. The value of two joint lives, both 40, is 9.82, by the ſame Table. The eſtate correſponding to 100*l.* is 4*l. per ann.* and the preſent value of ſuch an eſtate to be entered upon by a perſon 30 years of age, provided he ſurvives a perſon 40 years of age is, by the rule juſt quoted, *l.* 33.32. And this value, divided by 1 *l.* increaſed by its intereſt for a year, or by 1.04, is *l.* 32.03. the value in a *ſingle preſent payment* of the ſum of 100*l.* dependent on the given ſurvivorſhip. And this ſingle payment, divided by 10.43, is *l.* 3.07, the required value in *annual payments*, during the joint lives, if the firſt payment is not to be made till the end of a year. But if the firſt payment is to be made immediately, the required value in *annual payments* will be *l.* 32.03, divided by 11.43, or *l.* 2.8.—Theſe values, according to the *London* obſervations, or Mr. *Simpſon*'s Tables founded upon them, are *l.* 35.30, in a *ſingle payment*, and *l.* 3.6, in *annual payments*, beginning immediately.

Mr. *Simpſon*, in the Problems following that here quoted, has given ſolutions of moſt other Queſtions, concerning the values of reverſions depending on ſurvivorſhips, where the whole duration of two or three lives is concerned. And I am acquainted with no other ſolutions of theſe Queſtions, which are applicable to all Tables of obſervations, and which at the ſame time (proper regard being paid to the correction explained in the laſt Queſtion) may be conſidered as ſufficiently correct (*a*).

QUESTION XII.

" Suppoſe an inſtitution for the relief of
" widows to extend its aſſiſtance likewiſe
" to the families of married men, provided
" they leave no widows. Suppoſe, for in-
" ſtance, that in this caſe children are to be
" entitled to 100*l.* What is ſuch an expec-
" tation worth, in preſent payment, accord-
" ing to Dr. *Halley*'s Table, intereſt being at
" 4 *per cent?*"

ANSWER.

If 40 is the mean age at which members are admitted on ſuch an inſtitution, and 32 the mean age of their wives, the anſwer (ſuppoſing no ſubſequent marriages) is, by the 33d Problem in Mr. *Simpſon*'s Select Exerciſes, p. 298, and the correction already explained, *l.* 13.80 (*b*).

But

(*a*) See the third Eſſay.

(*b*) This Problem and its ſolution are given by Mr. *Simpſon* in the following words. " A and his heirs are " entitled to an eſtate of a given value, upon the deceaſe " of B, provided B ſurvives A; to find the value of " their expectation in *preſent* money."—Solution. "Find " the value of an annuity on the longeſt of two equal " lives, But there is a reduction neceſſary, on account of the chance there is, that a widower may marry again. Suppoſe, therefore, one half of all widowers to marry a ſecond and third time, and that two fifths of ſuch widowers ſurvive theſe ſubſequent marriages. In this caſe, $\frac{1}{2}$ added to $\frac{2}{5}$ of $\frac{1}{2}$, or $\frac{7}{10}$ of all who become widowers, will die without leaving widows, and therefore $\frac{7}{10}$ of *l.* 13.8, or *l.* 9.66, will be the anſwer. If only one *fourth* of all who become widowers marry again, and two fifths of theſe ſurvive, the anſwer will be *l.* 11.73.

“ lives, whereof the common age is that of the older of
“ the lives A and B; which value ſubtract from the
“ perpetuity, and take half the remainder; then it will
“ be as the expectation of duration of the younger of
“ the lives A and B, is to that of the older, ſo is the
“ ſaid half remainder to the number of years purchaſe
“ required, when the life B *is the older of the two*. But
“ if B *be the younger*; then to the number thus found,
“ add the value of an annuity on the longeſt of the lives
“ A and B, and ſubtract the ſum from the perpetuity, for
“ the anſwer in this caſe.”

If the eſtate is 4 *l. per annum*, the age of B 40, and of A 32, intereſt 4 *per cent.* the anſwer by this rule comes out *l.* 14.35, which divided (as in the preceding queſtion) by 104, gives *l.* 13.80, the value, as above, of 100*l.* in money. If B is 30 and A 40, the ſame value is 20*l.*

N. B. The value of the longeſt of two lives is always the *difference* between the value of the *joint* lives, and the *ſum* of the values of the two given *ſingle* lives. Thus; the value of a life at 40, is, by Table VI, 13.2. The *ſum* of the values of two ſuch lives, is 26.4. The value of two joint lives, whoſe common age is 40, is, by Table VII, 9.82; and the difference is 16.58, or the value of the *longeſt* of two lives at 40.

This calculation ſuppoſes all marriages to leave children who ſurvive their parents. If this is conſidered as uncertain, the values now determined muſt be diminiſhed in the proportion of this uncertainty.—Thus; if one marriage in ſeven fails of leaving children (*a*) that ſurvive their parents; theſe values will be reduced a *ſeventh* part, or to *l.* 8.28, if *half*, and *l.* 10.05, if a *quarter* of all widowers marry.

In this way may any other queſtions of the ſame kind be anſwered on any ſuppoſitions that may be thought moſt reaſonable.

QUESTION XIII.

" Let an eſtabliſhment be ſuppoſed which " takes in at once all the marriages in a " country, or all marriages among perſons " of a particular profeſſion within a given " diſtrict, and ſubjects them for perpetuity " to a certain equal and common tax, or an- " nual payments, in order to provide life an- " nuities for ſuch widows as ſhall reſult from " theſe marriages. What ought the tax to " be, ſuppoſing the annuity 20*l.* and calcu- " lating at 4 *per cent.* from Mr. *De Moivre*'s " valuation of lives; or, which is nearly the " ſame, from the probabilities of life in Dr. " *Halley*'s Table of obſervations?"

(*a*) This for many years has been nearly the fact among the miniſters and profeſſors in *Scotland*.

ANSWER.

ANSWER.

Since at the commencement of ſuch an eſtabliſhment, all the oldeſt, as well as the youngeſt marriages, are to be entitled equally to the propoſed benefit, a much greater number of annuitants will come immediately upon it, than would come upon any ſimilar eſtabliſhment, which limited itſelf in the admiſſion of members to perſons not exceeding a given age. This will check that accumulation of money, which ſhould take place at firſt, in order to produce an income equal to the diſburſements at the time when the number of annuitants comes to a *maximum*; and, therefore, will be a particular burden upon the eſtabliſhment in its infancy. For this, ſome compenſation muſt be provided; and the equitable method of providing it, is, by levying *fines* at the beginning of the eſtabliſhment, on every member *exceeding* a given age, proportioned to the number of years which he has lived beyond that age. But in the preſent queſtion, it is ſuppoſed, that ſuch fines cannot be conveniently levied, or that every payment muſt be equal and common, whatever diſparity there may be in the value of the expectations of different members. The fines, therefore, muſt be reduced to one common one, anſwering as nearly as poſſible to the diſadvantage I have mentioned, and payable by every member at the time when the eſtabliſhment begins. After this, the eſtabliſhment will be the ſame with one that takes upon it all at the time they marry; and the tax or annual payment of every member adequate to its ſupport, will be the annual payment during marriage, due from perſons who marry at the mean age at which, upon an average, all marriages may be conſidered as commencing.—There are then two points to be here determined. The *fines* neceſſary to be paid at firſt, according to the account I have juſt given; and the *conſtant annual payment*, neceſſary to be made by every member, as an equivalent for the expectation provided by the eſtabliſhment.—The *fines* to be paid at firſt are, for every particular member, the ſame with the difference between the value of the expectation to him at his preſent age, and what would have been its value to him had the ſcheme begun at the time he married? Or, they are, for the whole body of members, the difference between the value of the common expectation, to perſons at the mean age of all married perſons taken together as they exiſt in the world, and to perſons at that age, which is to be deemed their mean age when they marry.

Thus; let 33 for the man, and 25 for the woman, be the mean ages of all that marry annually. Let alſo 48 be the mean age of all the married men in the world, and 40 of married women (*a*).—Now, he that will calculate for theſe ages, in the manner directed in Queſt. IV. will find, that the value in *annual payments* during marriage, and beginning immediately, of the expectation of an annuity of 20*l.* *per annum*, by a perſon 25 years of age, after a life whoſe age is 33, is *l.* 6.64.—And that *l.* 8.04, is the value of the ſame expectation, the ages being 48 and 40.

The former, therefore, is the payment for perpetuity from every member of the eſtabliſhment; and the value of the *difference* between it and the latter, or of *l.* 1.4 *per ann.* payable during two joint lives, whoſe ages are 40 and 48, that is, *l.* 14.2, is the fine neceſſary to be levied on every married member at the beginning of the eſtabliſhment (*b*).

It would be eaſy to extend the benefit of ſuch an eſtabliſhment, ſo far as to provide 100*l.* for the children of members, provided

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(*a*) I muſt beg leave to refer to note (F) in the Appendix, for an explanation of what I mean by the mean ages of married men and women, and alſo for a confirmation of the anſwer I have given to this queſtion.

(*b*) An annuity for ever, the firſt payment of which is to be made immediately, is worth 26 years purchaſe, intereſt being at 4 *per cent.* *l.* 14.2 therefore, is equivalent in value to c.55*l.* or 11*s.* *per annum*, for ever. Add this to *l.* 6.64, and it will appear, that *l.* 7.19 *per annum*, beginning immediately, is the anſwer to this queſtion, ſuppoſing the value of the *fine* to be provided for in the perpetual annual payments.

they leave no widows; and the neceſſary addition on this account to the perpetual annual payments, can ſcarcely, in the circumſtances this queſtion ſuppoſes, be much more than about 15*s.* payable during life, and excluding from all benefit ſuch as happen to be widowers at the commencement of the eſtabliſhment, and do not afterwards marry.

If, in ſuch an eſtabliſhment, all perſons of a particular denomination, whether married men, widowers, or batchelors, are ſubjected alike to the taxes and fines; they ought to be as much *leſs*, as the whole number of perſons ſubjected to them, is *greater* than the number of marriages conſtantly exiſting.

In carrying theſe ſchemes into execution, there cannot be a more eaſy, or equitable way of raiſing the neceſſary fines, than by providing, that none ſhall be entitled to any expectation for a few of the firſt years. Thus; an eſtabliſhment, entitling widows to 20*l.* *per annum* for life, and conſiſting of 667 married members, and 344 unmarried, always kept up at an average, ought to begin with a capital of *l.* 14.2 multiplied by 667, or 9471*l.* beſides one payment in hand of the conſtant annual payments. That is, (the proper annual payment of every member being in this caſe $\frac{667}{1011}$, multiplied by *l.* 6.64, or *l.* 4.38) it ought to begin with a capital of 13,899*l.* over and above the payment of *l.*4.38, at the *end* of every year for ever afterwards (*a*). — The excluſion of all the firſt members from any benefit, unleſs they ſurvive the firſt *two* years, or live to make *three* payments, would raiſe this capital nearly. And ſuch an excluſion for *three* or *four* years, would be an advantage ſo conſiderable, that it would probably give ſecurity and ſtability to the ſcheme for all ſubſequent time.

In theſe obſervations, I have had in view, ſeveral ſchemes of the kind deſcribed in it, which are now actually eſtabliſhed in this kingdom; but more particularly, one begun among the *London* and *Middleſex* clergy, and another which is eſtabliſhed by act of parliament, among the clergy in *Scotland*, of both which, I ſhall have occaſion in the next chapter to take further notice.

I have choſen to calculate here only from Dr. *Halley*'s Table, or Mr. *De Moivre*'s *hypotheſis* grounded upon it, becauſe the *London* Table is, by no means, adapted to the caſes in view.

The difference of eight years between the ages of men and their wives, as here taken, is probably too little; and for this reaſon,

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(*a*) Or, ſuppoſing the value of 9471 *l.* (the fine) provided for in the annual payments, it ought to receive every year, at the *beginning* of the year, a contribution from each member of *l.*4.74.

and alſo on account of the greater mortality of males, the values I have given ſhould be conſidered as the loweſt that any ſcheme ought to provide.

It ſhould be further remembered, that when the mean ages, at which marriages commence, are ſuppoſed to be 33 and 25, all ſecond and third marriages are included; and that it is to be expected, that almoſt all theſe marriages will begin after theſe ages; and likewiſe, that a conſiderable proportion of the firſt marriages will begin a much longer time *after* theſe mean ages, than any of the other firſt marriages will begin before them.—Probably, therefore, theſe mean ages ſhould not be taken younger. One or two years, however, more or leſs, in every ſuppoſition I have made, will make no difference of any conſequence.

QUESTION XIV.

"A perſon of a given age has an eſtate de-
"pending on the continuance of his life for
"a given term. What ought he to give for
"having it *aſſured* to him for that term?"

ANSWER.

From the value of an annuity certain for the given term, found by Table II, ſubtract the value of the life for the given term, found by Queſt. VI. and *reſerve* the remainder.—Multiply the value of 1 *l.* due at the end of the given term, (found by Table I.) by the *perpetuity*, and alſo by the *probability*, that the given life ſhall fail in the given term. The *product* added to the *reſerved* remainder, and the *ſum* multiplied by the given annuity, will be the required value of the aſſurance in one preſent payment (*a*).

EXAMPLE.

An eſtate or annuity of 10*l.* *for ever*, will be loſt to the heirs of a perſon now 34, ſhould his life fail in 11 years. What ought he to give for the *aſſurance* of it for this term?—That is; what is the preſent value of ſuch an annuity to be entered upon at the failure of ſuch a life, ſhould that happen in 11 years?

The value of the life of a perſon whoſe age is 34 for 11 years, is, by Queſt. VI. (reckoning intereſt at 4 *per cent.* and calculating from Dr. *Halley*'s Table of obſervations) 7.76; which, ſubtracted from 8.760, (the value of an annuity certain for 11 years) leaves 1 *l.* the *remainder* to be reſerved.

The value of 1 *l.* to be received at the end of 11 years, is, 0.6496, by Table I. The probability that the life of a perſon, aged 34,

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(*a*) See the demonſtration in note (G) Appendix.

ſhall fail in 11 years, is, by Dr. *Halley*'s Table, $\frac{103}{499}$; and the perpetuity is 25. Theſe numbers, multiplied by one another, and 1 added to the product, make 4.34, which, multiplied by 10, (the given annuity) gives *l.* 43.4, the required value in a ſingle preſent payment.

l. 43.4, divided by 1.04, gives *l.* 41.7, the true value, by Scholium to Queſt. X. of the aſſurance of an *equivalent* ſum, or of 250*l.* for 11 years on the given life.

Again. 41.7, divided by 8.76, (the value of the given life for the given time with unity added to it) gives 4.76, the ſame value in annual payments beginning immediately, for 11 years (*a*), ſubject to failure ſhould the life fail.

SCHOLIUM.

In a ſimilar way may the price of aſſurances on any two joint lives, or the *longeſt* of two lives for any given terms, be calculated; the rule being as follows:

"From the value of an annuity certain "for the *given term*, ſubtract the value of "the joint lives, or the longeſt of the two "lives for the *given term*, found by Scholium to Queſt. VI. and *reſerve* the remainder.—Multiply the value of 1 *l.* to be re-

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(*a*) The laſt payment to be made at the end of the 11th year; or 12 payments in all.

" ceived at the end of the given term by the " perpetuity, and alſo by the probability " that the *joint lives*, or the *longeſt of the two* " *lives*, ſhall fail within the given term. This " product added to the reſerved *remainder*, " and the *ſum* multiplied by the annuity to be " aſſured, will be the value of the aſſurance " in a ſingle preſent payment."

EXAMPLE.

" What is the value of 10*l.* *per annum*, to " be entered upon, ſhould *either* of two " perſons, one 40 and the other 30 years of " age, die in ten years, reckoning intereſt " at 4 *per cent.* and calculating from Dr. " *Halley*'s Table."

The value of two joint lives at theſe ages, for 10 years, (found by *Scholium* to Queſt. VI.) is, 6.51; which, ſubtracted from 8.111, (the value of an annuity certain for 10 years, at 4 *per cent.*) leaves 1.60, the remainder to be *reſerved*.

The value of 1*l.* to be received at the end of 10 years, is, .6755, by Table I.

The probability, that the lives of one or other of two perſons, aged 30 and 40, ſhall fail in 10 years, is, $\frac{185}{531}$, by Table III (*a*).

(*a*) The probability taken from the Table, that a perſon aged 30, ſhall live 10 years, is, $\frac{445}{531}$. That a perſon, aged 40, ſhall live 10 years, is, $\frac{346}{445}$. That they ſhall *both* live 10 years, is, $\frac{346}{445}$, multiplied by $\frac{445}{531}$, or $\frac{346}{531}$. That they ſhall *not both* live 10 years, or that *one* or *other* of them ſhall die in this time, is, $\frac{346}{531}$, ſubtracted from unity, or, $\frac{185}{531}$. See note p. 23.

And the perpetuity 25. Theſe numbers, multiplied by one another, and 1.60 added to the product, make 7.48, which, multiplied by 10, (the given annuity) gives *l.* 74.8, the anſwer in a ſingle preſent payment.

l. 74.8, divided by 1.04, gives *l.* 71.92, the value of the aſſurance of an *equivalent ſum*; or of 250*l.*—*l.* 71.92, divided by 7.51, (the value of the two joint lives for 10 years with unity added) gives 9.57, the value of the ſame ſum in annual payments beginning immediately, for 10 years, ſubject to failure ſhould the joint lives fail.

EXAMPLE II.

"What is the value of 10*l. per annum*, to be "entered upon, ſhould two perſons one 30, "and the other 40, *both* die; that is, ſhould "the *longeſt* of the two lives fail in 10 years, "reckoning intereſt at 4 *per cent.* and cal-"culating from Dr. *Halley*'s Table?"

The value of the *longeſt* of the two lives for 10 years, (that is, the value of the joint lives for 10 years, ſubtracted from the ſum of the (*a*) values of the ſingle lives for 10 years) is, 7.91; which, ſubtracted from 8.111, the value of an annuity certain for 10 years, leaves .20 the remainder to be reſerved.— The value of 1*l.* to be received at the end

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(*a*) See Scholium to Queſt. VI.

of 10 years, is, .6755. The probability that the lives of two perſons, aged 30 and 40, ſhall fail in 10 years, is, by Table III, $\frac{86}{531}$, multiplied by $\frac{99}{445}$, or $\frac{8514}{236295}$; and the perpetuity 25. Theſe numbers, multiplied by one another, and .20 added to the product, make .740, which, multiplied by 10, (the given annuity) gives 7.4, the anſwer in a ſingle payment.

7.4, divided by 1.04, gives 1 1, the value of the aſſurance of 250*l.*

REMARK I.

The values of ſingle lives for given terms, when theſe terms are leſs than 10 years, muſt, in anſwering theſe Queſtions, and alſo in anſwering the following Queſtions, be found true to at leaſt 2 or 3 places of decimals. But they cannot be found to this exactneſs by any Tables that are extant; and, therefore, they muſt be calculated in the following manner:

"Multiply the probability, taken out of "the Table of obſervations, that the life "ſhall exiſt 1, 2, 3, &c. years, by the value "of 1*l.* due at the end of 1, 2, 3, &c. years; "and the ſum of the products will be the "value of the life for 1, 2, 3, &c. years."

For Example. The probability, that a perſon whoſe age is 34, ſhall live a year, is, by Dr. *Halley*'s Table, $\frac{490}{499}$. The probability, at the ſame age, of living 2 years, is, $\frac{481}{499}$; 3 years, $\frac{472}{499}$.—$\frac{490}{499}$ multiplied by .9615, (the value, by Table I, of 1 *l.* due at the end of a year, intereſt being at 4 *per cent.*) is, .942; or the value of the life for *one* year.—$\frac{481}{499}$, multiplied by .9245, (the value of 1 *l.* due at the end of 2 years) is, .891. And this added to the former product, gives 1.833; or the value of the life for 2 years.—$\frac{472}{499}$, multiplied by .8890, (the value of 1 *l.* due at the end of 3 years) is, .841; and this product, added to 1.833, makes 2.674, or the value of the given life for 3 years.

When the term exceeds 10 years, the rule in Queſt. VI. will give theſe values with ſufficient exactneſs; and it would do the ſame in all caſes, were the values of lives given true to 3 or 4 places of decimals, and in ſtrict agreement to the Tables of obſervation uſed.

The remark now made is to be extended to the values of *joint* lives for given terms. For theſe values, like thoſe of *ſingle* lives, cannot be found in ſolving theſe Queſtions with ſufficient accuracy, when the terms are ſmall, by any method, except the tedious one, of multiplying the probability that the 2 lives ſhall *both* continue 1, 2, 3, &c. years, by the value of 1 *l.* due at the end of 1, 2, 3, &c. years, and taking the ſum of the products in the manner juſt deſcribed.

REMARK II.

If the annuity is to be entered upon, in caſe of the failure within a given time of any life or lives, *at the end of that time*; and not *at the end of the year in which the failure may happen*; its preſent value will be the product ariſing from the continual multiplication by one another of the perpetuity increaſed by unity; the value of 1 *l.* due at the end of the given time; the annuity; and the probability that the life, or lives, ſhall fail within the given time. And care ſhould be taken not to confound theſe two ſorts of Queſtions with one another.—Thus; the value in one payment of 10*l.* *per ann.* to be entered upon eleven years hence, in caſe a perſon aged 34 ſhould not live ſo long, is 26, (the perpetuity increaſed by unity, intereſt being at 4 *per cent.*) multiplied by .6496, and by 10*l.* and alſo by $\frac{103}{499}$; or 34.8.—This value, divided by 1.04, is, 33.5, the value of an equivalent ſum, or of 250*l.* to be obtained on the ſame conditions.

The value of the *aſſurance* of any annuity on the whole continuance of any ſingle life is, by Queſt. X. the *exceſs* of the perpetuity above the value of the life, multiplied by the annuity. And in like manner; the value of the *aſſurance* of any annuity on the whole continuance of any two *joint* lives, or the *longeſt* of two lives, is the exceſs of the *perpetuity*

petuity above the value of the joint lives, or of the longeſt of two lives, multiplied by the annuity. This is very obvious; but no general method has been yet explained of finding the values of *aſſurances* on lives and ſurvivorſhips for terms of years leſs than the whole continuance of the lives. For this reaſon, I have been here more explicit than I ſhould otherwiſe have been; and, as ſuch aſſurances are now much practiſed, and may be very uſeful if their values are rightly determined, I have thought proper to add the two following Queſtions, which, when joined to Queſtion XI. and Mr. *Simpſon*'s 33d *Problem* given in the note p. 39, will, I believe, exhauſt this ſubject as far as two lives can be concerned.

QUESTION XV.

"B, expectant, will loſe a given ſum, "ſhould he ſurvive A, *within a given time.* "What ought he to pay for the *aſſurance* of "it?—In other words: "What ought he to "pay for a given ſum to be received at the "death of A, ſhould he happen to ſurvive "him within a given time?"

ANSWER.

Divide the *ſum* of the decrements of life in the Table of obſervations from the age of A, for the given time, by the given time; and, by the *quotient*, divide the number of the living in the Table at the age of A; and again, by this *ſecond* quotient (*a*), divide the given ſum reſerving the *third* quotient.

Find the value of an annuity on the life of B, for the given time. To this value add the *quotient*, that will ariſe from dividing the value of an annuity certain, for the given time, by twice the *complement* of the life of B; and the *ſum*, multiplied by the *reſerved quotient*, will be the required value in a ſingle preſent (*b*) payment.

EXAMPLE.

Let the Table of obſervations be Mr. *Simpſon*'s for *London*, or Table VIII. Let the rate of intereſt be 3 *per cent.* A, ſeven years of age. B, 30. The given time 14 years. The given ſum 100*l.*—The ſum of the *decrements*, in Table VIII. for 14 years from the age of ſeven, is 73, which, divided by 14, gives 5.2. The number of the living at ſeven is 430, which, divided by 5.2, and 100*l.* divided by the quotient, gives *l.* 1.21, the *quotient* to be *reſerved.*

(*a*) When the age of A is under 60, and the term ſo large as to exceed the difference between it and 70, it will be beſt, when the *London* Table is uſed, to divide the given ſum, not by the ſecond quotient here mentioned, but by the *complement* of the life of A, taken out of Table IX.

(*b*) See the demonſtration of this rule, and alſo of the rule that will be given for ſolving the next Queſtion, in the Appendix, note (H).

The value of an annuity for 14 years on the life of B, is, by Queſt. VI. 9.5.—The value of an annuity certain for 14 years, is, by Table II. 11.296, which, divided by 94.4, (twice the *complement* of the life of B, by Table IX) (*a*), gives .12, which, added to 9.5, gives 9.62; and this again multiplied by 1.21, the *reſerved quotient*, gives 11.64, the *preſent* value in *one* payment of 100*l.* payable at the death of A aged 37, to B aged 30, ſhould A die and leave B the ſurvivor within 14 years.

The preſent value for 14 years of two joint lives, one 7 and the other 30 years of age, may be found, by the help of Table XI, and the rule in the *Scholium* to Queſt. VI. to be nearly 9 years purchaſe; and, *l.* 11.64 divided by this value with unity added, or by 10, gives 1.164, the foregoing value in *annual payments* during the joint lives for 14 years, the *firſt* payment to be made immediately, and the *laſt* payment at the end of 14 years, ſhould the joint lives not fail.

SCHOLIUM.

It deſerves particularly to be remembered, that in this method likewiſe may be calculated, what ſums ought to be paid on any ſurvivorſhip, within a given time, of one life

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(*a*) This Table gives the *expectations* only, but it ſhould be remembered, that twice the *expectation* is always the *complement* of a life. See note, p. 37.

beyond another, in conſideration of any given ſum now advanced.—The following Example of this is a caſe which has offered itſelf in practice.

"A perſon, aged 30, has in expectation
" an eſtate which is to come to him, pro-
" vided he ſurvives a *minor*, aged 7, before
" he is out of his minority; that is, pro-
" vided he ſhould be himſelf living at the
" time of the minor's death, ſhould that hap-
" pen before he is 21.—In theſe circum-
" ſtances, he wants to borrow 1000*l.* on his
" *expectation*. What *reverſion* out of the
" eſtate depending on ſuch a ſurvivorſhip, is
" a proper equivalent for this ſum now ad-
" vanced, intereſt being reckoned at 3 *per*
" *cent.* and the probabilities of life being
" ſuppoſed the ſame with thoſe in Mr. *Simp-*
" *ſon's* Table of *London* obſervations?"

ANSWER.

It appears from what has been juſt determined, that for *l.* 11.64 now advanced, the proper equivalent in ſuch circumſtances, is, 100*l.* to be paid, in caſe the ſurvivorſhip ſhould take place; or, by the *correction* in page 34, as much of the eſtate as 100*l.* will buy at 3 *per cent.* ſuppoſing the firſt rent to be received immediately; (that is, ſuppoſing the eſtate worth 34.33 years purchaſe.) or *l.* 2.912 *per annum.*—By the rule of proportion, therefore, for 1000*l.* the proper equivalent will be 8591*l.* in money, or 250*l.* *per annum* out of the eſtate.

QUESTION XVI.

" 100*l.* will be loſt to B's heirs, ſhould he " happen to die after A, *within a given time*. " What is the price of the *aſſurance* of it?— " That is: What is the preſent value of " 100*l.* payable at the death of B, provided " his death ſhould happen *after* A's death, " *within a given time?*"

ANSWER.

Divide the ſum of the decrements of life in the Table of obſervations from the age of B, for the given time, by the given time; and by the *quotient* divide the number of the living at the age of B; and again, by this *ſecond quotient* (*a*), divide the given ſum, reſerving the *third quotient*.

Find the value of an annuity on the life A for a number of years, leſs by *one* year than the given time, which ſubtract from the value of an annuity certain for the ſame number of years. Multiply the *remainder* by the *reſerved quotient*, and divide the *product* by the amount of 1*l.* for one year, and let this be a *ſecond* reſerved quotient.

(*a*) Or rather, if the *London* Table is uſed, by the *complement* of the life of B, when his age is under 60, and the term exceeds the *difference* between it and 70.

Again. Multiply into one another the *firſt* reſerved quotient, and the value of an annuity certain for the given time; and divide the product by twice the *complement* of A's life. This *laſt* quotient, added to the *ſecond* reſerved quotient, will be the *anſwer* in a preſent ſingle payment.

EXAMPLE.

Let the age of B be 40. Of A 30. The ſum 100*l.* Rate of intereſt 4 *per cent.* The given time 20 years. The Table of obſervations, Mr. *Simpſon*'s, or Table VIII.—The ſum of the decrements of life, in this Table, from the age of 40 for 20 years, is 127, which, divided by 20, (the given time) gives 6.38.—The number of the living at 40 is 229, which, divided by 6.38, gives 35.8; and 100*l.* (the given ſum) divided by 35.8, gives 2.79, the *firſt* quotient to be reſerved.

The value of an annuity for 19 years on a life at 30 years of age, is 10.3; which, ſubtracted from 13.134, (the value of an annuity *certain* for 19 years, by Table II) and the remainder multiplied by 2.79, gives 7.89. This product divided by 1.04, (the amount of 1 *l.* in one year) gives 7.60; the *ſecond* reſerved quotient.

2.79 multiplied by 13.59, (the value of an annuity certain for 20 years) gives 37.916; and this *product* divided by 94.4, (twice the complement of A's life by Table IX.) gives .401, which, added to 7.60, gives 8*l.* the *Anſwer*; or, the value of 100*l.* payable at the death of B, on the contingency of his ſurviving A aged 30, and *both* dying in 20 years.

It is plain, that this is likewiſe the ſum that ought to be lent to B now, on the expectation of 100*l.* at his death, provided it ſhould happen after A's death in 20 years.

This rule gives the juſt ſolution in all caſes, except when B, the expectant, is the *youngeſt* of the two lives, and at the ſame time the term of years *greater* than the complement of A's life. In this particular caſe the following rule muſt be uſed.

Find, by the preceding rule, the value of the aſſurance of the given ſum for a term of years, equal to the complement of A's life, and let this value be *reſerved*. Multiply by one another the given *ſum*; the *value* of 1*l.* to be received at the end of a number of years equal to the complement of A's life; and the value of an *annuity certain* for as many years as the given term exceeds this complement. And the *product*, divided by the complement of B's life, and the *quotient* added to the *value reſerved*, will be the true value ſought.

EXAMPLE.

Let the age of B be 30; of A 40. The term 47 years; and every thing elſe as in

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the laſt Example. The complement of A's life, is, by Table IX, 39.2. The value of 100*l.* to be received at the death of B, if he ſurvives A within 39 years, may be found by the preceding rule to be *l.* 16.15; the value to be reſerved.—The value of 1 *l.* to be received at the end of 39 years is, by Table II, .2166. The value of an annuity *certain* for 8 years, (the exceſs of the given term above the complement of the life of B by Table IX.) is, 6.733.

And theſe two values multiplied by one another, and by 100*l.* give 145.83; which, divided by 47.2, (the complement of the life of B) and 16.15, added to the quotient, make *l.* 19.23, the value ſought.

REMARK.

As after finding the preſent value of an eſtate, or annuity, it is neceſſary to *divide* that value by the amount of 1 *l.* in one year, in order to find the preſent value of a *ſum* equivalent to the *annuity*; ſo, after finding the value of a ſum, it is neceſſary to *multiply* that value by the ſaid amount, in order to find from it the value of an equivalent annuity.

In the firſt Example, therefore, the value of an eſtate of 4*l.* *per annum*, would be *l.* 8.32. In the ſecond Example, 20*l.* And this is, as it ought to be, the value for the whole duration of the lives, agreeably to the Problem in the note page 37.

In ſolving this Queſtion, care alſo muſt be taken not to forget the *firſt* Remark under the foregoing Queſtion.

In the ſame way with that in which the rules in the three laſt Queſtions have been diſcovered, it is poſſible to find rules for calculating the values of *aſſurances, for given terms,* on lives and ſurvivorſhips, where three or more lives are concerned. But this is of leſs importance; and I chuſe to leave to others the further proſecution of this ſubject.

CHAP. II.

Containing an Application of the Queſtions in the foregoing Chapter to the Schemes of the Societies in Great Britain, *for making Aſſurances on Lives and Survivorſhips, and for granting Annuities to Widows, and to Perſons in old Age.*

SECT. I.

Of the London Annuity, *and the* Laudable *Societies for the Benefit of Widows.*

THE ſcheme mentioned in Queſt. VIII. is nearly that of the *London Annuity* Society. The *Laudable Society* is alſo formed on a ſimilar plan. In both, the *annual contribution* of every member is five guineas, payable half-yearly; and for this a title is given to an annuity of 20*l.* to every widow during widowhood, if the huſband, after admiſſion, lives *one* year according to the *firſt* ſcheme; or *three* years according to the (*a*) *ſecond*;

(*a*) In this ſociety a member who lives but *one* year, is entitled to no more than an annuity of 10*l.* for his widow; If he lives two years, 15*l.* If he lives three years, 20*l.* four years, 25*l.* ſeven years, 30*l.* ten years, 35*l.* thirteen years, 40*l.*

ſecond; of 30*l.* if the huſband lives *ſeven* years, according to both ſchemes; and 40*l.* according to the *firſt* ſcheme, if he lives 15 years, or 13 years, according to the ſecond. — In both ſchemes alſo, there is no other premium or fine required, than five guineas extraordinary, at admiſſion, from every member whoſe age does not exceed 45. The *Laudable Society* admits none above 45, and the *London Annuity* Society obliges every perſon between 45 and 55 to pay, at admiſſion, five guineas extraordinary, for every year that he is turned of 45.

Theſe are the main particulars in theſe ſchemes; and, therefore, both of them, were the annuities to be enjoyed for life, would receive (ſuppoſing the members all under 46 at admiſſion, and of the ſame ages with their wives, and money at 4 *per cent.*) but little more than three fifths of the true value of the annuities; or about one half, ſuppoſing wives, one with another, 10 years younger than their huſbands; as appears from Queſtion VIII.

It appears further in that Queſtion, that, ſuppoſing the annuities to be *life* annuities, and men and their wives of equal ages, the expectation to which an annual payment of five guineas beginning immediately, entitles, is nearly 14*l.* if the contributor lives a year, and 20*l.* if he lives ſeven years (*a*), taking

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(*a*) The ſame annual payment will, on the ſame ſuppoſitions, entitle to 14*l.* if a member lives a year, and 18*l.* if he lives *three* years.

the medium between the *London* and the other Tables of Obſervation.

It is likely, that many perſons will be very unwilling to believe, that theſe ſchemes are ſo deficient as they have been now repreſented. I will, therefore, endeavour to prove this in a way which, tho' leſs ſtrict, is ſufficiently deciſive, and may be more likely to be intelligible to perſons unſkilled in mathematical calculation.—I ſhall here confine myſelf to the ſcheme of the *London Annuity* Society. The differences between it and the ſcheme of the *Laudable* Society are inconſiderable, and what ſhall be ſaid of the one will be fully applicable to the other.

According to this ſcheme, as it has been juſt deſcribed, all that live 15 years in the ſociety will be entitled to annuities of 40*l.* *per annum* for their widows. Suppoſe the whole ſociety, at admiſſion, to be men of 40 years of age, taken one with another. A perſon of this age has an even chance of *living* 23 years; and he has an even chance of continuing with a wife of the ſame age, (that is, of continuing in the ſociety) 13 years and ½ (*a*). Not much leſs, therefore, than half

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(*a*) This is the exact truth according to Mr. *De Moivre*'s Hypotheſis, and the *Norwich* Table. But according to Dr. *Halley*'s and the *Northampton* Table, a man 40 years of age has an even chance of living no more than 22 years, and of joint continuance with a wife of the ſame half the members will continue in the ſociety 15 years; and, conſequently, not much leſs than half the widows that will come upon the ſociety will be annuitants of 40*l.* *per annum.* Theſe widows, however, being older than the reſt when they commence annuitants, will continue on the ſociety a ſhorter time; and, therefore, the number conſtantly in life together, to which they will in a courſe of years increaſe, will be proportionably ſmaller. Putting every thing as favourably as poſſible, let us ſuppoſe, that out of 20 annuitants conſtantly on the ſociety, *five* will be annuitants of 40*l.* *ſix* of 30*l.* and *nine* of 20*l.* To 20 annuitants then the ſociety will pay 560*l.* *per annum*, or the 20th part of this ſum, that is 28*l.* to *every* annuitant at an average. But ſuch an annuity for a life at 40, after another equal life, provided both ſurvive one year, is worth (by Queſt. VII. p. 24.) in a ſingle preſent payment, 85*l.* nearly, according to the *London*, and all the Tables of Obſervations, intereſt being all along ſuppoſed at 4 *per cent*.

It cannot appear improbable to any one, that this ſhould be the true value of ſuch a reverſion. It is not credible, that there is

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ſame age, 13 years.—Forty muſt be more than the mean age of the members of the ſociety at admiſſion, and on this account the number of annuitants of 40*l.* muſt be proportionably greater. The mean age, therefore, has been taken very moderately.

any ſituation in which the decrements of life are ſuch as can make it a tenth part more or leſs.—85*l.* in preſent payment is the ſame with 3*l.* 8*s.* *per annum* for ever.—But is an annual payment of five guineas, which muſt ceaſe as ſoon as either of two lives each 40, fails, equal in value to ſuch a perpetuity? Every one muſt ſee, that there is a great difference.—A ſet of marriages between perſons all 40, will, according to the probabilities of life in Dr. *Halley*'s Table, laſt, one with another, 15 years (*a*); and an annual payment beginning immediately, during the joint continuance of two perſons of this age, is worth 10 years purchaſe (*b*). The compariſon then, in the preſent caſe, is between 3*l.* 8*s.* *per annum for ever*, and five guineas *per annum for* 15 *years*; or between an annuity of 3*l.* 8*s.* worth 25 years purchaſe, and an annuity of five guineas worth only 10 years purchaſe.

But to throw this ſubject into another light.

(*a*) See the beginning of Eſſay I.

(*b*) The value of ſuch an annual payment, by Table XI, or the *London* Obſervations, is 9.1; and 10.8, by Mr. *De Moivre*'s Hypotheſis.—I have not taken into this account the five guineas *fine* paid at admiſſion, becauſe it is obviouſly of too little conſequence to make any conſiderable difference. The allowances I have made in favour of theſe ſchemes are more than equivalent to it. In particular; it ſhould be remembered, that the payments required by theſe ſchemes, are *half-yearly payments* beginning immediately; and that theſe, by Queſt. VIII. are leſs advantageous than the payments I all along ſuppoſe them to require, or, "*yearly payments* beginning immediately."

Let the number to which the ſociety is kept up be ſuppoſed to be 200. It has been demonſtrated in Queſt. II, that at leaſt half this number of widows will in time come to be conſtantly on the ſociety; and it has alſo been juſt now ſhewn, that the medium of annuities, payable to them, will be at leaſt 28*l.* After a courſe of years, then, the ſociety will have a conſtant expence to bear of 2800*l. per annum.*—But what will be its income?—In order to determine this, we muſt conſider, that there are two ſources from whence its income will be derived. Firſt, the annual payments of the members. And, ſecondly, the money accumulated, or the *capital* raiſed during the time the number of annuitants is coming to a *maximum.*—The firſt of theſe ſources affords 1000 guineas, or 1050*l. per annum.* This wants 1750*l.* of the annual expence juſt mentioned; and, therefore, in order to have the income of the ſociety equal to the burden upon it, when the annuitants come to a *maximum,* there muſt be a fund raiſed in the mean time equal to 43,750*l.* or to an eſtate in perpetuity of 1750*l. per annum.*—But 1050*l. per annum* beginning immediately, and forborn 24 years, and improved, without loſs or delay, all that time at 4 *per cent.* compound intereſt, will but juſt raiſe ſuch a capital (*a*). There is, therefore,

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(*a*) Every Queſtion of this kind may be eaſily ſolved in the following manner. In Table I, find the value of fore, the fulleſt proof, that the ſcheme I am conſidering is extremely deficient. The truth is, that ſcarcely a *third* of ſuch a capital could be raiſed, as will appear from the following obſervations.

Out of 200 perſons, all 40 years of age, *more* than five, according to the *London* Table of Obſervations, and not ſo many by Dr. *Halley*'s Table, may be expected to die in a year. Suppoſe then five to be the real number of members that will die the firſt year of the ſociety. In ſubſequent years the collective body of members will be continually growing older; and, therefore, the proportion of them that will die every year, will be continually increaſing, till it gets to a *maximum*. I will, however, ſuppoſe, that during the firſt 20 years no more than the

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of 1 *l.* payable at the end of any number of years; and any given annuity divided by that value, will be the annuity to which the given annuity will in that number of years increaſe. — Thus; the preſent value of 1 *l.* payable at the end of 25 years, is .3751, reckoning intereſt at 4 *per cent.* and 1050*l.* *per annum* divided by .3751, gives 2,800*l.* *per annum*, the increaſed annuity ariſing from 1050*l.* *per ann.* In the ſame manner; it may be found, that the ſame annuity, forborn 11 years, will increaſe to 1610*l.* *per annum.* This ſuppoſes the firſt payment of the annuity to be made a year hence. If the firſt payment is to be made immediately, which is the preſent caſe, the annuity will increaſe to the ſame ſums in one year leſs time.—But a more particular account of this will be given in the rules annexed to the Tables at the end of this work.

number juſt ſpecified will die every year; and that, conſequently, no more than *five widows* will come every year on the ſociety. The ages of all theſe widows, when they commence widowhood, will, it is evident, be between 40 and 60. One with another then, they may be conſidered as having commenced widowhood at 50 years of age. Now, five widows left every year at this age, will, in 10 years, increaſe to 43 conſtantly in life together, according to the expectations of life in Tables III, IV, and V; and, in 20 years, to 70 (*a*). Suppoſe the true number alive together at the end of 20 years to be only 62. the greater part of theſe will be annuitants of 30*l.* and 40*l.* *per ann.* and the reſt 20*l.* Were the former only equal to the latter, the medium of annuities payable to them would exceed 25*l.* Suppoſe then this medium to be no more than 26*l.* and it will

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(*a*) Every calculation of this kind is eaſily made by the rule in note (A) in the Appendix.—I have put the number living together at the end of 20 years at 62, not only that the reader may be better ſatisfied that I have kept low enough, but alſo to make an allowance for ſuch widows as will be left by thoſe members who die within a year after admiſſion, and who, therefore, according to theſe ſchemes, will be entitled to no annuities. This allowance is too large: For, after the firſt year of the ſcheme, it will not happen above once in 4 or 5 years, that the death of a member will be ſo circumſtanced, ſuppoſing the probability that a man at 40 will live a year, to be, as all but the *London* Tables make it, 50 to 1.

follow, that, at the end of 20 years, the ſociety will have an annual rent to pay of 26*l.* multiplied by 62 or 1612*l.* and, if then able to bear ſuch an expence, it muſt, in the intermediate time, have acquired an increaſe of income equal to the difference between 1050*l.* and 1612*l. per ann.* That is; it muſt, with its ſavings, have accumulated a ſtock equal to 562*l. per ann.* and worth 14,050*l.* But, as during this time, there will be a number of annuitants conſtantly increaſing, to whom yearly payments muſt be made, the ſavings of the ſociety cannot certainly be one half of what they would have been had it been all the time free from all burdens. Suppoſe then the ſtock produced by theſe ſavings, to be equal to the ſtock that would ariſe from an income of 1050*l. per ann.* beginning immediately, and improved perfectly at 4 *per cent.* compound intereſt, for half the time I have mentioned, or for 10 years, without being ſubject to any checks or deductions. Such an income thus improved, would, in 10 years, produce an additional income of 560*l. per annum*, or a capital of 14,000*l.*—According to theſe obſervations, therefore, the annual income of the ſociety at the end of 20 years, and before a third part of the higheſt annuitants could come upon it, would begin to fall ſhort of its expences. About that time then it would neceſſarily run aground; and long before the number of annuitants could riſe to a 100, it would ſpend its whole ſtock, and find itſelf under a neceſſity of either doubling the annual payments of its members, or of reducing the annuities one half.

All I have now ſaid is meant on the ſuppoſition, that the ſociety begins with 200 members at 40 years of age, and is afterwards limited to that number, by admitting no more new members than will juſt ſupply the vacancies occaſioned by the loſs of old members. If it is allowed to increaſe, it may continue a longer time. And, for this reaſon, a ſociety that wants half the income neceſſary to render it permanent, may very well ſubſiſt, and even proſper for 30 or 40 years.—Thus, the *Laudable Society*, was it to keep to its preſent number of members, might poſſibly feel no deficiencies for 20 or 30 years to come; but if it ſhould continue to increaſe at the rate of 70 or 80 every year, it would, at the end of that time, poſſeſs a balance ſo much in its favour, as might enable it to ſupport itſelf for 20 or 30 years more (*a*). But bankruptcy would come

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(*a*) What has been before demonſtrated in Queſt. III. ſhould be here recollected, that the number of annuitants on ſuch a ſociety as this, muſt go on to increaſe for more than 100 years, after acquiring its greateſt number of members.

The *Laudable* Society, I am informed, took its riſe from a calculation contained in a pamphlet entitled, *The Poſſibility and Probability of a* SCHEME *intended for the Benefit of Widows being able to ſupport itſelf.* The ſcheme here referred to, is the ſame with that which this Society has ſince come at laſt, and with the more terrible weight the longer it had been deferred.

The rule in the *London Annuity* Society, which obliges every perſon between the ages of 45 and 55, to pay at admiſſion 5 guineas extraordinary, for every year that he exceeds 45, is an advantage to it, but it is a very inadequate, and alſo a very unequitable advantage. For at the ſame time, that it obliges a perſon 55 years of age, to give *more* than the value of his expectation, it takes *above* two fifths *leſs* than the value from a perſon who is 45 years of age.

Should any perſons remain ſtill doubtful about what I have ſaid, I would beg them to attend to one further argument.

It muſt be reckoned upon that every other member of theſe ſocieties, ſuppoſing them to conſiſt of perſons all of the ſame ages with their wives, will leave widows to whom one with another, (as already ſhewn) at leaſt 28*l.*

ſince followed; and I am afraid I ſhall not be credited, when I ſay, that the calculation to prove its capacity of ſupporting itſelf, is founded on the ſuppoſition, that a hundred married men whoſe common age is 36, will leave but *one* widow every year, tho' at the ſame time it is ſuppoſed that two of them will die every year.

This miſtake has made the whole calculation one half wrong.—Nothing can be plainer than that, if the death of a married man does not leave a widow at the end of the year, the reaſon muſt be, that both himſelf and his wife have happened to die in the year. But it is always very improbable this ſhould happen.

per ann. muſt be allowed, for as many years as there have been payments from each member. For every 10 guineas then received they muſt ſome time or other hereafter pay 28 *l.* But let it be well conſidered what can enable them to do this. Did money bear no intereſt, for any given ſum now received, they could not afford at any time hereafter to pay more than an *equal* ſum. That is; (ſince the duration of *ſurvivorſhip* is in the preſent caſe, by Queſt. II, equal to the duration of *marriage*) the proper conſideration for any given reverſionary annuity, to be allowed to *all* the ſurvivors of a ſet of marriages, would be, ſuppoſing no intereſt of money, an equal annuity payable by each marriage during its exiſtence; and juſt *half* the reverſionary annuity, if it is to be allowed only to half the ſurvivors, or to widows excluſive of widowers. The annual payment then of *five* guineas, during marriage, can entitle widows to no more than an annuity of *ten* guineas, ſuppoſing money to bear no intereſt. But if it does bear intereſt, the ſame payment will entitle them to more, in proportion to the degree in which it is capable of being improved, during the time between that in which the annual payments begin, and the commencement of widowhood. Now, it is eaſy to ſee, that unleſs money bears very high intereſt, this improvement cannot be likely in any circumſtances to produce a capital, the intereſt of which ſhall be equal to the annual payment itſelf. Any given annual payment perfectly improved at 4 *per cent.* compound intereſt, requires 17 years to double itſelf, ſuppoſing the firſt payment made immediately; or, near 18 years (*a*), if the firſt payment is not made till the end of a year. But no marriages are *likely* to laſt ſo long as this, except thoſe among perſons who are very young. A marriage between two perſons, both 40, will not *probably* laſt longer than 13 years, according to the probabilities of life in Dr. *Halley*'s Table. A marriage between two perſons, both 50, will not probably, by the ſame Table, laſt longer than *eleven* years; nor a marriage between two perſons, both 30, longer than 16 years. Such marriages, it is true, may poſſibly laſt 30 or 40 years. But this circumſtance is more than balanced by the fact, that no leſs poſſibly they may not laſt *one* year. The annual payments, then, being incapable of ſuch an improvement as ſhall produce an additional income equal to themſelves; it is obvious, that no ſociety ought to go ſo far as to allow to widows annuities twice as great as thoſe which might be allowed, ſuppoſing no

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(*a*) At 3 *per cent.* the period of doubling money by compound intereſt, is nearly 23 years. At 5 *per cent.* 14 years.

intereſt of money (*a*); ſo far, for inſtance, as to allow, inſtead of 10 guineas, 20 guineas for an annual payment of five guineas. In the circumſtances of moſt of theſe ſocieties three fifths addition may be the full allowance. That is; ſuppoſing the annual payment of each member to be five guineas, time may be expected for gaining from hence a capital of 75 guineas, or that ſhall produce three guineas *per annum* intereſt; and the proper reverſionary annuity will be 16 guineas; or ſix guineas more than the proper reverſionary annuity, did money admit of no improvement.

The preceding obſervations have gone on the ſuppoſition, that the reverſionary annuities are to be *for life*. What difference in favour of theſe ſocieties ariſes from the circumſtance, that the annuities are to be paid only *for widowhood*, cannot be exactly determined. Some judgment, however, may be formed of it from what has been ſaid at the concluſion of Queſt. II. Were even one half

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(*a*) The money accumulated will not be exactly the ſame with that to which the annual payment would increaſe, if improved at compound intereſt for a number of years, equal to that which the joint lives have an *equal chance* of exiſting. Much leſs will the increaſe be the ſame with that which would ariſe from the annual payment forborn, and improved for a number of years equal to the *expectations* of the joint lives. It will be leſs than either of theſe, for a reaſon explained in note (L) Appendix.

of the widows to marry, ſtill the ſchemes I have been conſidering would probably be inſufficient. But in the circumſtances of theſe ſocieties it cannot be expected, that above one in 10, or perhaps one in 20, will marry. The perſons moſt likely to enter into them, are ſuch as have not the proſpect or ability of making competent proviſions for their widows in other ways. The widows left, therefore, will in general be unprovided for, and being alſo left with families of children, it is quite unreaſonable to expect, that any conſiderable proportion ſhould marry. This is true of ſuch as may happen to be left young; but when a ſociety has ſubſiſted ſome time, the *greater* part will not be young when left, and theſe, at the ſame time that no advantage can be expected from their marrying, will be in general the *higheſt* annuitants, and, therefore, the *heavieſt burdens*.—Moreover, the proſpect of the loſs of their annuities will have a particular tendency to check marriage among them.—For all theſe reaſons it ſeems to me likely, that the benefit, which theſe ſocieties will derive from marriage among their annuitants, will not be very conſiderable; or at leaſt not *ſo* conſiderable as to be equal to the advantages I have allowed them, by calculating on the ſuppoſitions, that the money they receive will be *always improved perfectly, without loſs or delay, at the rate of* 4 *per cent. compound intereſt*; that the probabilities of life among males and females are the ſame, and all huſbands likewiſe of the ſame ages with their wives, and that conſequently the *maximum* of widows on ſuch ſocieties can amount to no more than half the number of marriages (*a*). — With reſpect to the laſt of theſe ſuppoſitions, it deſerves to be particularly obſerved, that by an enquiry made ſome years ago in *Scotland*, it was found, that the widows of *miniſters* and *profeſſors* there, (*b*) notwithſtanding the diminution occaſioned by their marrying, did exceed half the number of marriages. And certainly it would be unreaſonable in theſe ſocieties not to reckon that the ſame will happen among them. — Indeed it ſeems certain that, notwithſtanding

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(*a*) Care ſhould be taken in theſe ſocieties, not to judge of the proportion of widows that will marry, from the proportion that may happen to marry during their firſt years. For moſt of the widows that will be left at firſt will be young; whereas the greater part will not be young when they commence widowhood, after a ſociety has ſubſiſted 30 or 40 years; and, therefore, tho' one in 3 or 4 ſhould marry at firſt, it will not be reaſonable to expect, that half ſo many ſhould marry after the affairs of the ſociety become ſtationary.

(*b*) 364 widows, all living at one time, were counted; and the number of married miniſters and profeſſors for many years paſt has been, at an average, 667.—Twenty widows likewiſe are left one year with another; and, for 10 years, ending in the year 1767, but nine of theſe had married.—Of the annuitants likewiſe (about 160 in number) on the fund eſtabliſhed among the Diſſenters in *London*, for relieving the widows of indigent miniſters, it is found that few ever marry. See the latter end of the 4th Eſſay; and note (A) in the Appendix.

the hazards that attend child-bearing, the probability, that the woman ſhall ſurvive in marriage, and not the man, is much greater (*a*) than is commonly imagined. It will be ſhewn in the laſt Eſſay, that it is not leſs than the odds of 3 to 2; and had I calculated agreeably to this fact, the values of annuities for widows, would have been given near a quarter greater than they have been given on the ſuppoſition, that the chance of ſurvivorſhip is equal between men and their wives.—It muſt be added, that I have made no account of any expences attending the execution and management of the ſchemes of theſe ſocieties. Some ſuch expences there muſt be, and ſome advantages ſhould be always provided in order to compenſate them.

There are in this kingdom ſeveral inſtitutions for the benefit of widows, beſides the two on which I have now remarked; and in general, as far as I have had any information concerning them, they are founded on plans

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(*a*) Partly, as obſerved in page 8, on account of the greater mortality of males, but chiefly on account of the exceſs of age on the man's ſide.—According to the printed articles of agreement, the *Laudable Society* pays no regard to this exceſs of age; and the allowance required on this account by the *London Annuity* Society is ſo trifling that it deſerves no notice.

In March 1770, thirty-two huſbands had died in the *Laudable Society*, and 27 wives. They ſeem, therefore, to be already beginning to experience, that the chances of ſurvivorſhip in marriage are in favour of the wife.

equally inadequate. The motives which influence the contrivers of theſe inſtitutions are, without doubt, *laudable*; but they ought, I think, to have informed themſelves better. This appears ſufficiently from what has been ſaid; but I will juſt mention one further proof of it.

The *London Annuity Society* promiſes that, if in 21 years; and the *Laudable Society* that, if in 25 years, it ſhall appear that there has been all along an annual ſurplus in favour of the ſocieties, it ſhall be employed in either raiſing the *annuities*, or in ſinking the *annual payments*. Now, they may be aſſured, that, if at the end of theſe periods, they ſhould not be poſſeſſed of a conſiderable ſurplus, the true reaſon will be, their having granted much higher annuities than the annual contributions are able permanently to ſupport: For it has been demonſtrated, that the number of annuitants, and conſequently the amount of the annual expences, will go increaſing for a long courſe of years beyond theſe periods. The effect, therefore, of carrying into execution this regulation will be, precipitating that bankruptcy which would have come too ſoon had there been no ſuch regulation.

It has been ſaid in defence of theſe Societies, that the deficiencies in their plans cannot be of much conſequence, becauſe their rules oblige them to preſerve a conſtant equality between their income and expences, by reducing the annuities as there ſhall be occaſion.

And

And from hence it is inferred, that they can never be in any danger of a bankruptcy.—In anſwer to this, it has appeared, that the time when they will begin to feel deficiencies is ſo diſtant, that it will be too late to remedy paſt errors, without ſinking the annuities ſo much, as to render them inconſiderable and trifling. All that is given too much to *preſent* annuitants is ſo much taken away from *future* annuitants. And if a ſcheme is *very* deficient, the firſt annuitants may, for 30 or 40 years, receive ſo much more than they ought to receive, as to leave little or nothing for any who come after them. Deficient ſchemes, therefore, are attended with particular injuſtice; and this injuſtice will be the ſame, if, inſtead of *reducing* the annuities, the annual payments ſhould be increaſed; for all the difference this can make will be, to cauſe the injuſtice to fall on *future contributors*, inſtead of *future annuitants*.

But what requires moſt to be conſidered here is, that, after either the annuities have been for ſome time in a ſtate of reduction, or the contributions in a ſtate of increaſe, it will be ſeen that theſe Societies have gone upon wrong plans, and, therefore, they will be deſerted and avoided; the conſequence of which will prove ſtill greater deficiencies in their annual income, and a more rapid deſertion and decline, 'till a total diſſolution and bankruptcy take place.—This will be the death of moſt of the preſent ſocieties for providing for widows, if they continue to be encouraged, and do not ſoon alter their plans: And at that period the number of *annuitants* will be greater than ever; whoſe annuities, having no other ſupport than the poor remains of a ſtock always inſufficient, will be ſoon left, without the poſſibility of relief, to lament that ignorance and credulity which gave riſe to theſe ſocieties, and which had ſo long ſupported them.

In the *London* Annuity Society, there is an encouragement to *batchelors* and *widowers* to join them, ariſing from the additional annuities to which they will be *immediately* entitled, when they marry, in conſequence of having made their payments a greater number of years; and it is imagined, that particular advantages will be derived from ſuch members. But even theſe will in general pay much leſs than the value of their expectations.—A perſon who begins an annual contribution of five guineas at the age of 24 will, ſhould he live 11 years, and marry a woman of the ſame age at the end of that time, entitle her immediately to 35*l.* *per ann.* during ſurvivorſhip, and to 41*l.* *per annum* ſhould he live four years after marrying. (intereſt being at 4 *per cent.*) (*a*). In this par-

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(*a*) The value of five guineas *per annum* (firſt payment made immediately) for 11 years, ſubject to failure ſhould a life now 24 fail; and, after 11 years, for the joint lives of particular caſe, therefore, a perſon will pay nearly the true value of his expectation. But *all* at all ages who *marry*; and *moſt* of thoſe who *die*, in leſs time than 11 years after admiſſion, will pay leſs than the value of their expectations.

SECT. II.

Of the Aſſociation among the London *Clergy, and the Miniſters in* Scotland, *for providing Annuities for their Widows.*

IN April, 1765, the clergy within the bills of mortality, and the county of *Middleſex*, at a general meeting in *Sion-College*, agreed to form themſelves into a ſociety for the ſupport of their widows and orphans. Many in this reſpectable body may be capable of doing, in a better manner, what I have attempted in this Treatiſe; and they are, perhaps, already ſenſible of the deficiencies in the plan which they have eſtabliſhed. I ſhall not, however, I hope, do wrong, in taking the liberty to recite briefly this plan, in order to introduce a few obſervations upon it.

of two perſons both 35, is, by the Table of *London Obſervations*, *l.* 69.3.—By Dr. *Halley*'s Table *l.* 76.44.—The preſent value of 35 *l. per annum* for life to the widow of a perſon now 24, ſhould he live 11 years, and marry a woman of the ſame age with himſelf at the end of that time; and alſo of 6 *l.* more, or 41 *l. per annum* in all, ſhould he live after marriage four years; is, by the Table of *London* Obſervations, *l.* 69.36,—By Dr. *Halley*'s Table, *l.* 76.03.

According to the printed articles, every clergyman poſſeſſed of any benefice, lectureſhip, or licenſed curacy, within the bills of mortality, and the county of *Middleſex*, who ſubſcribes annually one guinea, or two guineas, or more, ſhall entitle his widow to an annuity; or, if he leaves no widow, he ſhall entitle any ſuch children as he ſhall leave, to the ſame annuity for ſeven years as his widow would have had. And, in caſe a widow poſſeſſed of an annuity, ſhould either *die* or *marry* before the lapſe of 10 years, from the commencement of her annuity, ſuch children of her former huſband, as ſhall be then alive, are to be entitled to as many of the ten years payments of the annuities as ſhe ſhall not have received.—The annuity is fixed to no particular ſum, but inſtead of this, it is ordered, that a fourth part of the annual ſubſcriptions and intereſt ſhall be divided the firſt three years after the eſtabliſhment of the ſociety; half only the next four years; and 3-4ths the next 5 years; provided, however, that in no one of theſe 12 years the dividend ſhall exceed 20*l.* to the widows and orphans of the clergy ſubſcribing two guineas or more; and 10*l.* to the widows and orphans of the ſubſcribers of one guinea. And, after the expiration of 12 years, the whole amount of the ſubſcriptions, and of the intereſt of the capital ſtock, is to be divided proportionably for ever.—It is further provided, that every clergyman, who ſhall be married, or have children,

dren, at the time of his ſubſcription, ſhall pay a fine of two guineas towards a capital ſtock, if a ſubſcriber of two guineas or more, and 40 years of age or upwards. If 50 years of age or upwards, he ſhall pay a fine of three guineas; if 60 or upwards, five guineas. But, if not married at the time of his ſubſcribing, and ſhall afterwards marry, he ſhall pay a fine according to the age he ſhall be of at the time of his marrying. The obligation laid upon all, whether married or unmarried, to become ſubſcribers, is, an incapacity of being admitted members without the conſent of a general court, unleſs, within two years after becoming poſſeſſed of any eccleſiaſtical employment, they ſubſcribe.

Every one who has attended to the obſervations in this and the preceding chapter, muſt know what judgment to form of theſe regulations.

Let us ſuppoſe, that all the clergy in *London* and *Middleſex* came into this aſſociation from the firſt; and that one with another they are ſubſcribers of two guineas annually; and that there are among them as many unmarried perſons as married.

In this caſe, it may be learnt from Queſt. XIII, that the annuity to which widows ſhould be entitled, (ſuppoſing no allowance to the children of any that die) ought not to exceed 10 or 11 guineas at moſt, and that, beſides the annual ſubſcriptions, there ought to have been a fine paid at the commencement of the ſcheme, by every married perſon, of ſix guineas at leaſt, or, by the whole number of ſubſcribers, three guineas. If the number of married members is double the unmarried, the annuity ought not to exceed eight guineas; and the fine from every member ſhould be about four guineas.—The order, that only a fourth part of the annual ſubſcriptions and intereſt ſhall be divided the firſt three years, half the next four years, and three-quarters the next five, is without reaſon; becauſe the number of claimants, for the firſt 12 years of the ſcheme, will be ſo few, that it will not be poſſible, during that time, that there ſhould be occaſion for dividing any proportions ſo large of the annual ſubſcriptions and intereſt, unleſs they are indeed beyond all bounds too little. —After 12 years, the number of annuitants will go on increaſing for near 50 years, as appears from Queſt. III. The conſequence, therefore, of dividing, after this time, the whole amount of the annual ſubſcriptions and intereſt, will be a conſtant yearly diminution in the dividends for near 50 years; and making the payments to the firſt claimants much more conſiderable than they ought to be, at the expence of all ſubſequent claimants.—For theſe reaſons; it appears to me out of all doubt, that this ſcheme is by no means likely to anſwer the good ends propoſed by it; and that, therefore, it will be beſt to lay it aſide.

At the time it was ſettled it was, I find, further agreed, that the annual ſubſcriptions of the *laity*, together with the intereſt of their benefactions, unleſs otherwiſe directed by the donors; and the annual ſubſcriptions of ſuch of the clergy as ſhall ſo direct, ſhall make a *charitable fund* to be applied to the relief of the diſtreſſed widows or children of all the clergy within the limits I have mentioned, whether ſubſcribers or not, provided that in no one year of the firſt twelve more than 20*l.* be given out of the fund to any one family.—This is an excellent deſign; and if the money ariſing from all the ſubſcriptions is thrown into this fund, an important means of relief may be provided, for ſuch of the more indigent widows and families as will accept the help of charity.

There is one more aſſociation of particular conſequence, which it is neceſſary I ſhould take notice of. I mean, the *aſſociation* among the miniſters and profeſſors in *Scotland*, for making proviſions for their widows and orphans. The laſt-mentioned aſſociation, and alſo ſeveral others of the ſame kind (*a*) in this kingdom, have been formed on the

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(*a*) There is one among the Diſſenting Miniſters in the counties of *Cheſter* and *Lancaſhire*, and another among the Diſſenting Miniſters in *Cumberland*, *Northumberland*, *Weſtmoreland*, and *Durham*.—Even the *London Annuity* Society, tho' its plan is totally different, profeſſes to form itſelf on the principles of the *Scotch* eſtabliſhment, and to derive encouragement from it.

model of this eſtabliſhment; and the ſucceſs with which it has been hitherto attended, has been the principal cauſe to which they owe their riſe. — I am afraid of being too tedious, and therefore I will not attempt to recite all the particulars contained in the plan of this eſtabliſhment. It may be ſufficient to obſerve, that for "an annual "payment, which begun immediately, of "five guineas from 1011 contributors, 667 "of whom are married perſons; beſides a "tax on weddings producing about 142*l.* "*per annum*; it entitles every widow to an "annuity of 20*l.* during widowhood; and al"ſo, every family of children that ſhall be "left by ſuch members as die without leav"ing widows, to 200*l.*" Now, by particular enquiry at the commencement of the ſcheme, it had been found, that there was reaſon to believe that, for many years back, 20 widows had been left annually by the whole body of miniſters and profeſſors; and that, alſo, ſix had died annually and left children without widows; and theſe facts have been ſince confirmed by the experience of 25 years.—Subſtract, therefore, from 5,450*l.* (the whole annual income) 1,200*l.* payable every year to ſix families of children; and 4,250*l.* *per ann.* (the firſt payment of which was made (*a*) immediately) or *l.* 4.2 *per annum*

(*a*) The truth is, that a double payment was made at the beginning of the ſecond year. This is of leſs value than two *annum* from each member, will remain as the ſtanding proviſion for bearing the burden of the annuitants.—This proviſion, according to the calculation in Queſt. XIII, and note (F) Appendix, ought to be at leaſt 4,745*l. per annum*, from each member; from whence it ſeems to follow, that this eſtabliſhment has not a ſufficient income to afford it a *permanent* ſupport.—But I do not by any means deſign to aſſert this. The difference between the real and calculated incomes, as it has been now ſtated, is not conſiderable enough to give ſufficient reaſon for ſuch an aſſertion. This eſtabliſhment may have ſome advantages that I know nothing of, and that are not mentioned in the printed accounts; or, in conſequence of the increaſe of luxury, and the higher price of all the means of ſubſiſtence, marriage may decline among the miniſters; or, poſſibly, the probabilities of life among them, tho' much higher 'till towards 50, than is common among mankind in towns, may yet afterwards decreaſe much faſter. The income, therefore, of this eſtabliſhment, properly improved, may continue to be adequate to all its expences and burdens.—One obſervation, however, ought to be carefully attended to. The ſucceſs it has hitherto met with, is no good reaſon for en-

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two payments, one of which is made immediately, and the other a year hence; but, the difference not being conſiderable, I have reckoned them the ſame.

tertaining this expectation, with any degree of confidence. It appears from Queſt. III, that the number of annuitants, on ſuch an eſtabliſhment, muſt go on increaſing for 60 years, from the time of its commencement; and it is obvious, that the *continuance* of a ſucceſs which has not laſted *half* this period, cannot be abſolutely depended on. I know, indeed, that, according to the calculations which were made when this eſtabliſhment begun (*a*), the number of widows upon it will not increaſe ſenſibly after the year 1780, or for more than 10 years to come; and, were this true, all diffidence about it would, perhaps, be unreaſonable. But theſe calculations cannot, in this inſtance, be right; for they imply, that none are left widows under 52 years of age. 'Tis certain, on the contrary, that many are left widows under 32; and that, conſequently, the whole body in life together muſt go on to increaſe for 25 years, beyond the period aſſigned in theſe calculations; or till the year 1805 (*b*). It is neceſſary

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(*a*) See Table III, in a book printed at Edinburgh in 1748, entitled, Calculations, with the Principles and Data on which they are inſtituted, relative to a late act of parliament, entitled, *An act for raiſing and eſtabliſhing a fund for a proviſion for the widows and children of the miniſters of the church, and of the heads, principals and maſters of the univerſities of* Scotland; ſhewing the riſe and progreſs of the fund.

(*b*) This is ſaid on the ſuppoſition, that all the miniſters and profeſſors acceded to the ſcheme from the firſt.

ceſſary I ſhould add, that the whole number alive, when the increaſe ſtops, will, moſt probably, be greater than the number provided for in theſe calculations. They are made on the ſuppoſition, that 52 is the mean age at which women commence widows. If this ſuppoſition is right, it is impoſſible that, according to the probabilities of life in Dr. *Halley*'s Table, the number of widows living together at one time, derived from 20 left annually, ſhould increaſe to more than 323, if none marry; or about 300, if one marries every year (*a*): Nor does it appear likely, that this

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135 did not accede; but all their ſucceſſors have been obliged to accede. This circumſtance muſt add 30 or 35 years to the period of increaſe which I have mentioned; that is, as many years as are neceſſary to cauſe all the non-contributors to die off. See Queſt. III.

(*a*) The *expectation* of a perſon at the age of 52 is, when taken exactly from Dr. *Halley*'s Table, 16.16; and this number multiplied by 20, gives the *maximum*, to which 20 widows left annually, at 52, will increaſe in 34 years, ſuppoſing 86 the utmoſt limit of life. Vid. Eſſay I.—In the calculation to which I have referred, there is no account taken of thoſe that die in the year in which they are left widows; and, for this reaſon, they are made to increaſe to a greater number than is conſiſtent with the ſuppoſition, that 52 is their mean age when they commence widowhood.—It ſhould be remembered here, that ſuppoſing this mean age, as explained in note (F) Appendix, rightly taken, the *maximum* of widows will be rightly found in the manner juſt ſpecified. But the period, in which they would attain to that *maximum*, would be as many years greater than the difference between the mean age and the utmoſt limit of life, as the mean age is greater than this eſtabliſhment will be able to bear the expence of above 30 or 40 more than the laſt-

than the *leaſt* age at which widowhood ever commences. The calculation, therefore, which I have in view muſt be very wrong. It ſuppoſes not only, that all the widows left at all ages, will increaſe to a *maximum* in the ſame time with thoſe left at the mean age; but that all left in the courſe of every year will certainly live to the beginning of the next year. It ſuppoſes, likewiſe, that no widows will marry; and thoſe concerned will underſtand me when I add, that it ſuppoſes further, that *acceders*, tho' moſtly young miniſters unmarried, will leave widows as faſt as the *noncontributors* whom they ſucceed. In conſequence of theſe omiſſions it has all along given the numbers in life much higher than they ought to have been given; but yet the event has been, that theſe numbers have in fact correſponded nearly to the calculation: From whence it follows, that the widows in life have hitherto increaſed at a much greater rate than they could have done, according to Dr. *Halley*'s Table, had their mean age, when left, been 52. Either, then, their mean age has been conſiderably leſs than 52; or, their probabilities of life muſt be conſiderably greater than thoſe in Dr. *Halley*'s Table; and, it ought, therefore, to be expected, that they will at laſt increaſe to higher numbers than thoſe aſſigned in this calculation.

I cannot help here mentioning one more reaſon for entertaining this expectation.—Were the decrements of life *uniform*, and the chances of ſurvivorſhip between men and their wives *equal*, the number of widows and widowers in the world, if none married, would alſo be equal; and both together equal to the whole number of marriages. See Queſt. II, and note (F) Appendix.

If the chances for the ſurvivorſhip of the wife are greateſt, the number of widows in the world will be alſo greateſt, and the whole number of widows and widowers greater than the whole number of marriages. In the preſent caſe experience proves, that the chances of ſurvivorſhip in marriage are as 5 to 3 at leaſt, in favour of the laſt-mentioned number. But it was found, by enquiry, that the number of widows living

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the wife; or that there are 20 widows left to 12 widowers. (See note (F) Appendix, and the end of the laſt Eſſay). The number of widows and widowers then would certainly, if none married, exceed 667, the whole number of marriages. Suppoſe, however, that they would be only equal; which is the ſame with ſuppoſing, that *widows* would not increaſe to more in life together, in proportion to the number left, than *widowers*; or that one with another they are of equal ages. 667 then being the number of marriages, this will likewiſe be the number of widows and widowers, 416 of whom muſt be widows and 250 widowers; that is, 5 to 3. Now as *widows* are certainly, one with another, younger than *widowers*; and likewiſe, very probably, more long-lived at equal ages; and as alſo, in the preſent caſe, but *one* marries of the *twenty* left annually; it follows, with demonſtrative evidence, that if the annuitants on this eſtabliſhment ſhould not increaſe to 400, the reaſon muſt be, that the decreaſe in the probabilities of life, inſtead of continuing always uniform, is ſlow in the firſt ſtages of mature life, and accelerated afterwards, to a degree of which there is no example in any Tables. And this, poſſibly, may be indeed the caſe; for it is uncertain whether, in this particular ſituation, life may not waſte according to a law not yet obſerved. This uncertainty it is in the power of the conductors of this ſcheme to remove, by keeping an account of the ages at which all the miniſters and profeſſors enter upon their offices and die; and alſo of the ages of their wives when they marry and die. From ſuch an account kept for a courſe of years, Tables of Obſervation, adapted to the beſt ſort of lives of both ſexes, might be formed, which would contribute much to the improvement of this part of knowledge.

I have ſaid nothing above of the advantage which this eſtabliſhment derives from the marriages among widows. This advantage, it has appeared, were it enjoyed without abatement, would not be conſiderable; but it is in ſome meaſure given up by the order which makes a part of this eſtaing in 1748, was *at leaſt* 364; (*a*) and it may be learnt from notes (A and F) in the *Appendix*, that, according to Dr. *Halley's* Table, 20 widows left annually, muſt in time increaſe to near 400, tho' one of the youngeſt married every year. And it may be further learnt from note (A) Appendix, that the widows on this eſtabliſhment have not hitherto increaſed more ſlowly, than is conſiſtent with their actually increaſing to 400: Nor, indeed, (as the probabilities of life in this caſe are higher than thoſe given by Dr. *Halley's* Table) ſhould I much doubt of their increaſing to *more* than this number, were it not that 364 has been given as the number found upon enquiry. With reſpect to this, however, it ought to be mentioned, that another account had been taken which made the number of widows only 321. As, therefore, a more careful enquiry diſcovered 43 new widows; perhaps, an enquiry yet more careful would have made yet further diſcoveries. In taking ſuch accounts, none can well be added;

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eſtabliſhment, that ſuch children of an annuitant as are under 16, ſhall be entitled, if ſhe marries, to as much as ſhall happen to be then unpaid of ten years purchaſe of her annuity. The ſame proviſion is made for the children of annuitants that die. There are other *burdens* on this eſtabliſhment, and it has alſo *advantages* of which I have taken no notice; but, as far as I am acquainted with them, they are of no particular conſequence, and they alſo nearly balance one another.

(*a*) See Calculations, with their Principles and Data, &c. Introduction.

but in the ſtricteſt ſearch it can hardly be poſſible to avoid omitting ſome.

The reſult of the laſt enquiry, in particular, is ſaid to have been; not that it was found that there were *no more* than 364 widows, but that they did *exceed* this number. (*a*) However, let 364 be the true number living in 1748. Before that time, there had probably been more marriages among them, than there have been ſince; and this may have rendered their number leſs than it would otherwiſe have been, and leſs than it will be hereafter: For it ſeems to me, that this eſtabliſhment, at the ſame time that it encourages marriage among the miniſters, has a tendency to check it among their widows, by making the conſequence of marrying to be the loſs of their annuities.

I hope the venerable miniſters and profeſſors concerned in theſe remarks, and at preſent ſo eminent in all the departments of ſcience, will excuſe what has been now ſaid. It may, perhaps, be of ſervice, if not to them, yet at leaſt to ſome in this part of the united kingdoms, by ſhewing them, that this eſtabliſhment has been copied in it much too raſhly; and that, however ſucceſsful it may in the end prove, it is yet too ſoon to make it a model and an authority for ſimilar eſtabliſhments.

(*a*) See Calculations with their Principles, &c. p. 44.

SECT. III.

Of the beſt Schemes for providing Annuities for Widows.

INſtitutions for providing widows with annuities would, without doubt, be extremely uſeful, could ſuch be contrived as would be *durable*, and at the ſame time *eaſy* and *encouraging*. The natures of things do not admit of this in the degree that is commonly imagined. The calculations and rules, in the preceding chapter, will enable any one to determine in all caſes to what reverſionary annuities any given payments entitle, according to any given valuation of lives or rate of intereſt. From Queſt. VII and VIII, in particular, it may be inferred that (intereſt being at 4 *per cent.* and the probabilities of life as in Mr. *De Moivre*'s Hypotheſis, or the *Breſlaw*, *Norwich*, and *Northampton* Tables) for an annual payment beginning immediately of *four* guineas during marriage; and alſo for a guinea and half in hand, on account of each year that the age of the huſband exceeds the age of the wife, every married man, under 40, might be entitled to an annuity, during life, for his widow of 5*l.* if he lives a year, 10*l.* if he lives *three* years, and 20*l.* if he lives *ſeven* years. Money can ſcarcely now in this kingdom be improved at ſo high a rate as 4 *per cent.* But, perhaps, it might be reaſonably expected, that an advantage, ſufficient to compenſate this diſadvantage, would be derived, from changing the annuities I have mentioned into annuities during widowhood. One may, at leaſt, venture to pronounce, that nothing much worſe could befall a ſociety that went on this plan, than the neceſſity of ſome time or other adding half a guinea to the annual payments.

If ſuch a ſociety chuſes, that thoſe who ſhall happen to continue members the longeſt time, ſhall be intitled to ſtill greater annuities, ſix guineas, additional to all the other payments at admiſſion, would be the full payment for an annuity of 25 *l.* and 12 guineas for an annuity of 30 *l.* if a member ſhould live 15 years.

All batchelors and widowers might be encouraged to join ſuch a ſociety, by admitting them on the following terms.—*Four guineas* to be paid on admiſſion, and *three guineas* every year afterwards, during celibacy; and, on marriage, the ſame payments with thoſe made by perſons admitted after marriage; in conſideration of which, 1 *l. per annum*, for every ſingle payment before marriage, might be added to the annuities, to which ſuch members would have been otherwiſe entitled.

For example. If they have been members four years, or made five payments before marriage, inſtead of being entitled to life-annuities for their widows of only 5*l.* 10*l.* 20*l.* 25*l.* and 30*l.* on the conditions I have ſpecified, they might be entitled to annuities of 10*l.* 15*l.* 25*l.* 30*l.* and 35*l.* Or, if they have been members nine years, and made 10 payments, they might, inſtead of the ſame annuities, be entitled to annuities of 15*l.* 20*l.* 30*l.* 35*l.* and 40*l.*—In this caſe, the contributions of ſuch members as ſhould happen to deſert, or die in celibacy, would be ſo much profit to the ſociety, tending to give it more ſtrength and ſecurity.

This is one of the beſt ſchemes that I am able to think of, or would chuſe to recommend. There are, however, others no leſs ſafe and encouraging which ſome may prefer, and which therefore, I will juſt propoſe.

Let the probabilities of life be the ſame with thoſe in the Tables juſt mentioned. Let money be ſuppoſed to be improved at no higher intereſt than 3 *per cent.* Let the reverſionary annuities promiſed to widows be 10*l.* *for life*, if a member lives five years after admiſſion; and 15*l.* more, or 25*l.* in all, if he lives 11 years. The proper payments for ſuch an expectation, from married men not exceeding 50 years of age, will, in the neareſt and moſt convenient round ſums, be four guineas *in annual payments* beginning immediately, and two guineas in hand for every year that his age exceeds his wife's, not admitting any greater exceſs than 15 years: Or, if the whole value is given in *one preſent payment*, 40*l.* added to a guinea, for every year that his age falls ſhort of 50, beſides the payment juſt mentioned on account of diſparity of age.—For example. Four guineas in annual payments, beſides 10 or 20 guineas in hand, according as the age of the huſband exceeds the wife's 5 or 10 years. Or, if the whole value of the expectation is given in one payment, 10 guineas added to 40*l.* (that is 50*l.* 10*s.*) from a man whoſe age is 40; and, in like manner, 20 guineas added to 40*l.* (that is 61*l.*) from a man whoſe age is 30; beſides the payment juſt mentioned on account of diſparity of age.

If money is improved at 4 *per cent.* or, on account of any advantages attending a ſcheme, may be juſtly conſidered as ſo improved, the full payments for the expectation I have mentioned will be about one eighth, or half a guinea, leſs in the *annual payments* during marriage; and a quarter leſs in all the other payments. That is: A married man, *at* or *under* 50, would, beſides three guineas and half in annual payments during marriage, be bound to add a guinea and half for every year he is older than his wife: Or, if he

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chuſes to give the value of his expectation in one payment; beſides the common contribution of 30 *l.* and a guinea and half for every year his age exceeds his wife's; he would be bound to pay three quarters of a guinea, for every year he is leſs than 50 years of age; that is, 53 *l.* 12 *s.* 6 *d.* in all, ſuppoſing him 40 years of age, and 10 years older than his wife.—All theſe payments doubled would entitle to double annuities.

There is one particular advantage which ſocieties formed on a plan of this kind would enjoy (*a*)—Perſons who know themſelves ſubject to diſorders, which are likely to render them ſhort-lived, will have no great temptations to endeavour to gain admiſſion into ſuch ſocieties; and, if admitted, the danger from them will be leſs than on any other plan. Were it not for this danger, one might recommend the following plan, as one of the moſt inviting.

In the plans hitherto mentioned it is implied, that, if either a member or his wife dies within any of the periods ſpecified, the additional annuities, that would otherwiſe have become due, will be loſt. But it would be much more agreeable to a purchaſer, that they ſhould be made certain to his wife, provided ſhe lives to the end of theſe periods,

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(*a*) See another advantage mentioned under Queſt. VIII, p. 28.

tho' in the mean time his own life ſhould fail. The value of ſuch annuities may be computed by the rule in Queſt. IX.

Suppoſe, for inſtance, the *ſcheme* to be "that a wife ſhall be entitled certainly to a "life-annuity of 20*l.* the firſt payment of "which ſhall be made at the end of 12 years, "provided ſhe ſhould be then alive, and her "huſband dead; or at the end of any year "beyond this term in which ſhe may happen to be left a widow." Suppoſe it alſo ſtipulated, "that ſhe ſhall be entitled to "10*l.* more, or 30*l.* in all, on the ſame "terms, provided ſhe ſhould live 16 years." —The value of ſuch an expectation (intereſt being at 3 *per cent.* and the probabilities of life as in Mr. *De Moivre*'s Hypotheſis) will be, in the moſt convenient round ſums, ſuppoſing none admitted above 50 years of age, ſeven guineas in annual payments to be continued during marriage, and to begin immediately; beſides four guineas in preſent money for every year, as far as 15 years, that the huſband's age exceeds the wife's, if he is between 40 and 50, and three guineas on the ſame account if he is under 40: Or, if the whole value of the expectation is given in one preſent payment, 70*l.* added to a guinea and half, for every year that the huſband's age falls ſhort of 50, beſides the payment juſt mentioned on account of diſparity of age.

If the annuities are made to be annuities during *widowhood*, and not during *life*, and the advantage ariſing from hence, is ſuppoſed equivalent to the difference between the improvement of money at 4 *per cent.* and its real improvement; the value of the expectation juſt mentioned, (that is, its value at 4 *per cent.*) will be ſix guineas in annual payments; beſides three guineas in preſent money, for every year that the huſband's age exceeds the wife's, if he is between 40 and 50; and 2 guineas, if he is under 40: Or, if the whole value of the expectation is given in one preſent payment, 56*l.* added to 1*l.* 5*s.* for every year that his age falls ſhort of 50, beſides the payment laſt mentioned on account of inequality of age. (*a*)

(*a*) Suppoſing 16 years the only term, the annuity 20*l.* and intereſt at 4 *per cent.* the proper payments will be nearly, in the caſe of equal ages and *ſingle* payments, 46*l.*—40*l.*—29*l.* as the age of the man is 30, 40, or 50. Or, in *annual* payments, *l.*3.80.—*l.*3.66.—*l.*3.13.—Suppoſing the woman's age 10 years leſs than the man's, the ſame values will be, in *ſingle* payments, *l.*58.92.—*l.*56.56.—*l.*53.66.—In *annual* payments *l.*4.63.—*l.*5.—*l.*5.41.—It appears, therefore, that a ſociety, ſuppoſing money improved at the rate of 4 *per cent.* might entitle all married men *indiſcriminately*, who are under 50 years of age, to ſuch an expectation as this for their wives, for either 60*l.* in *one* payment, or five guineas in *annual* payments. — But equity requires, that different payments ſhould be made, according to the different comparative ages of men and their wives; and Tables might be formed for ſhewing, at one view, what theſe different payments ought to be in all caſes. If ſuch Tables are wanting, recourſe muſt be had to ſome ſuch eaſy rules as thoſe I have ſtated above.

He that will give himſelf the trouble to calculate, agreeably to the directions in the Queſtions to which I have referred, will find that, taking all particular caſes together, the rules now given come as near the truth as there is reaſon to deſire in an affair of this nature, the *defects* in ſome caſes being nearly compenſated by the *exceſſes* in others.

I have calculated here, as well as in moſt other places, from Mr. *De Moivre*'s *Hypotheſis*, becauſe its conformity to the three Tables which I have ſo often mentioned, convinces me, that it gives a proper *medium* between the different values of *town* and *country* lives. In the country the probabilities of life are much higher; but in *London*, and probably in all *great* towns and ſome *ſmaller* ones, they are much lower.

It is proper to add, that, according to the values of lives and ſurvivorſhips deduced both from the *London* and Dr. *Halley*'s Table, and taking intereſt as low as 3 *per cent.* all women whoſe huſbands are under 50 years of age, might be entitled to an annuity of 24*l.* during *life* (the firſt payment to be made at the end of the year in which they ſhall be left widows) for the ſum of 100*l.* ſuppoſing 3*l.* additional given on account of every year that they are younger than their huſbands.—At 4 *per cent.* an annuity of 30*l.* might be granted on the ſame terms.

In the year 1690, the company of *Mercers* in *London,* adopted ſuch a ſcheme as that laſt mentioned. For 100*l.* in *one preſent payment*, they entitled every ſubſcriber to a *life annuity* for his widow of 30*l.*; and this, at that time, (when money bore 6 *per cent.* intereſt) was conſiderably leſs than the value of the money advanced, ſuppoſing men and their wives of equal ages. As the intereſt of money ſunk, they ſunk alſo the annuity, firſt to 25*l.* and then to 20*l.* and 15*l.* But at laſt, after carrying on the ſcheme for above 50 years, finding the burden of the annuitants too heavy, and likely to go on increaſing, they were obliged to drop the ſcheme and to ſtop payment. In a little time, however, by a parliamentary aid of 3000*l. per ann.* which they are now enjoying, they were reſtored to a capacity of making good all their engagements, and of paying their arrears.—Their failure, is, indeed, much to be lamented; for, in conſequence of it, the public has loſt the benefit of an inſtitution, that for many years promiſed the happieſt effects, by encouraging marriage, and affording relief to indigence. The rapid fall of the intereſt of money; their admitting purchaſers at too advanced ages; and, particularly, their paying no regard to the difference of age between huſbands and their wives, muſt have contributed much to hurt them. Some of the principal cauſes, therefore, which have rendered them unſucceſsful, may be now avoided; and for this reaſon I ſhould be glad to ſee ſome ſimilar ſcheme, providing, as this did, annuities for *life*, and not for *widowhood*, undertaken. If well planned it would, I think, be a proper object of *parliamentary* encouragement.

It muſt, however, be remembered, that the iſſue of the beſt ſchemes of this kind muſt be in ſome degree uncertain. For want of proper obſervations, it is not poſſible to determine what allowances ought to be made, on account of the higher probabilities of life among females than males. No prudence can prevent all loſſes in the improvement of money; nor can any care guard againſt the inconveniencies to ſuch ſchemes, which muſt ariſe from thoſe perſons being moſt ready to fly to them who, by reaſon of concealed diſorders, feel themſelves moſt likely to want the benefit of them.

The ſocieties, therefore, on which I have remarked in the firſt Section of this chapter, would have reaſon to take warning from what has happened to the *Mercer's Company*, were the ſchemes on which they are formed perfectly unexceptionable. But I have demonſtrated that theſe ſchemes are very defective; and that the longer they are carried on, the more miſchief they muſt produce. 'Tis vain (as appears from Queſt. III.) to form ſuch eſtabliſhments with the expectation of ſee-ing their fate determined ſoon by experience. If not more extravagant than any ignorance can well make them, they *will* go on proſperouſly for 20 or 30 years; and, if at all tolerable, they *may* ſupport themſelves for 50 or 60 years; and at laſt end in diſtreſs and ruin. No experiments, therefore, of this ſort ſhould be tried haſtily. An unſucceſsful experiment muſt be productive of very pernicious effects. All inadequate ſchemes lay the foundation of *preſent* relief on *future* calamity, and afford aſſiſtance to a *few* by diſappointing and oppreſſing *multitudes*.

As the perſons who conduct theſe ſchemes can mean nothing but the advantage of the public, they ought to liſten to theſe obſervations. At preſent their plans are capable of being reformed; but they cannot continue ſo always; for the greater number of exorbitant payments they now make to annuitants, the more they conſume the property of future annuitants, and the leſs practicable a retreat is rendered to a rational and equitable and permanent plan (*a*). They ſhould, therefore, *immediately* (*b*) either *reduce*

(*a*) See p. 82, 83. Sect. I.

(*b*) Thus; was the *London* Annuity Society to make their loweſt annuity 10*l.* the next 20*l.* and the higheſt 30*l.* they would probably be ſafe. But, after proceeding on their preſent plan ſome years longer, ſuch a reduction would by no means be ſufficient.

their ſchemes, or change them into one of thoſe which I have propoſed. But, I am afraid, this is not to be expected. The neglect with which they have received ſome remonſtrances that have been already made to them, gives reaſon to fear, that what has been now ſaid will be in vain; and that thoſe who are to come after them, muſt be left to *rue* the conſequences of their miſtakes.

SECT. IV.

Of Schemes for providing Annuities for Old Age.

A General diſpoſition has lately ſhewn itſelf, to encourage ſchemes for granting *annuities* to perſons in the latter ſtages of life; and this has occaſioned the 6th Queſtion in the former Chapter; and, as a further and more particular direction in caſes of this kind, I have thought it neceſſary here to give the following Table.

Values of 1*l. per ann.* for life, after 50, to perſons whoſe ages are	Values in *one preſent payment*, intereſt 4 *per cent.*	Intereſt 3 *per cent.*	Values in *annual payments*, till 50, to begin at the end of a year, intereſt 4 *pr ct.*	Intereſt 3 *per cent.*
10	1.235	2.015	.0789	.113
15	1.583	2.444	.106	.146
20	2.028	2.989	.146	.193
25	2.594	3.644	.203	.259
30	3.369	4.508	.297	.366
35	4.446	5.667	.466	.559
40	5.953	7.232	.822	.950
Values of the ſame annuity, after 55, to ages				
30	2.114	2.937	.167	.211
35	2.722	3.632	.241	.297
40	3.732	4.708	.394	.464
45	5.088	6.115	.703	.803
Values of the ſame annuity after 60, to ages				
35	1.667	2.290	.135	.168
40	2.234	2.923	.203	.245
45	3.043	3.811	.327	.384
50	4.255	5.061	.600	.679

The numbers in the 2d and 3d columns of this Table, multiplied by any annuity, will give the value of that annuity in a *ſingle* payment, to be enjoyed for life, by the ages correſponding to thoſe numbers in the firſt column, *after* the age mentioned at the head of that column.—And in the ſame manner; the numbers in the 4th and 5th columns will give the values in *annual* payments.—Thus. The value of 44*l. per annum,* to be enjoyed for life, after 50, by a perſon now 40, (intereſt at 4 *per cent.*) is 5.95, multiplied by 44, or *l.* 261.9, in a *ſingle* payment; and .822, multiplied by 44, or *l.* 36.16, in *annual* payments till 50, the firſt payment to be made at the end of a year.

In order to find the ſame values, partly in *annual payments*, and partly in any given *entrance* or *admiſſion-money*; ſay; "As the value of the *given annuity* in a *ſingle* payment, "(found in the way juſt mentioned) is to the "given *entrance-money*; ſo is its value in *annual* payments, to a fourth proportional; "which, ſubtracted from the value in *annual* "*payments*, the *remainder* will be the annual "payment due, over and above the given entrance-money."

EXAMPLE.

Suppoſe a perſon now 40, to be willing to pay 200*l.* entrance-money, *beſides* ſuch an annual payment for 10 years as ſhall, together with his entrance-money, be ſufficient to entitle him to a life annuity of 44*l.* after 50. What ought the annual payment to be?

ANSWER.

L.8.55.—For, *l.*261.9, is to 200*l.* as *l.*36.16, to *l.*27.61; which, ſubtracted from *l.*36.16, the remainder is *l.*8.55.

This Table has been calculated from the *probabilities* and *values* of lives in Tables III. and VI. The probabilities of life among the inhabitants of *London*, are (as I have often had occaſion to obſerve) much lower than among the generality of mankind; and the values in the preceding Table, had they been given agreeably to the *London* Obſervations, would have been leſs. But, certainly, an office or ſociety, that means to be a permanent advantage to the public, ought always to take higher rather than lower values, for the ſake of rendering itſelf more ſecure, and gaining ſome *profits* to balance *loſſes* and *expences*.

There have lately been eſtabliſhed, in *London*, ſeveral ſocieties for granting ſuch annuities as thoſe now mentioned; and he that will compare their true values, as they may be learnt from the preceding Table, with the *terms* of admiſſion into theſe ſocieties, as given in their printed *Abſtracts* and *Tables*,

Tables, muſt be ſurpriſed and ſhocked. They are all impoſitions on the public, proceeding from ignorance, and encouraged by credulity and folly.

It has been ſhewn; that the proper payment, (allowing compound intereſt at 4 *per cent.*) for an annuity of 44*l.* to be enjoyed by a perſon now 40, for what may happen to remain of his life after 50, is 200*l.* in *admiſſion-money*; beſides *l.*8.55, or 8*l.* 11*s.*, in annual payments till he attains to 50, the firſt of theſe payments to be made at the end of a year.—The conditions of obtaining this annuity, according to the Tables of the *Laudable Society of Annuitants for the benefit of age*, are 76*l.* 17*s.* in *admiſſion-money*; and 6*l.* 14*s.* in *annual payments*.—According to the Tables of the ſociety of *London Annuitants for the benefit of age*, the conditions of obtaining the ſame annuity are 30*l.* in *admiſſion-money*, and 10*l.* in *annual payments*.—The *Equitable* Society of Annuitants requires for the ſame annuity 38*l.* 10*s.* in *admiſſion-money*, and 13*l.* in *annual* payments. The true value is, over and above the *admiſſion-money* juſt mentioned, an *annual payment* of 30*l.* 17*s.* (intereſt reckoned at 4 *per cent.*) or an *annual payment* of 36*l.* 15*s.* intereſt reckoned at 3 *per cent.*—The *London Union Society for the comfortable ſupport of aged members* promiſes an annuity of no leſs than 50 guineas for life, after 50, to a perſon now 40 for 40*l.* 10*s.* in admiſſion-money, and 7*l.* in annual payments.

The *Amicable Society of Annuitants for the benefit of age*, promiſes an annuity of 26*l. per annum*, for life, to a perſon now 40, after attaining to 50, for 28*l.* 16*s.* in *admiſſion-money*, and 6*l.* in *annual payments.*—The true value of this annuity is 28*l.* 16*s.* in *admiſſion-money*, and 17*l.* 8*s.* in *annual payments*, (intereſt ſuppoſed at 4 *per cent.*); or the ſame ſum in *admiſſion-money*, and 20*l.* 18*s.* in *annual payments*, intereſt ſuppoſed at 3 *per cent.*

The *Provident Society for the benefit of age* promiſes an annuity of 25*l.* to a perſon now 40, after attaining to 50, for 34 guineas in admiſſion-money, and eight guineas in annual payments. The true value is, 34 guineas in *admiſſion-money*, and 15*l.* 12*s.* in *annual* payments, intereſt at 4 *per cent.*; or, the ſame ſum in *admiſſion-money*, and 19*l.* in *annual* payments, intereſt being at 3 *per cent* (*a*).

But I will not tire the reader, by going, in this manner, thro' the ſchemes of all theſe ſocieties. The contrivers of them, it is certain, can know nothing of the principles on which the rule in Queſt. VI, and the demonſtration of it in the *Appendix* is founded; and, therefore, if unwilling to be guided by the autho-

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(*a*) The account here given of the terms on which a perſon whoſe age is 40, is admitted into theſe ſocieties, I have taken from their printed Tables as they ſtood at the end of the year 1770.—In the younger ages the deficiencies are greater.

rity of mathematicians, it may not be poſſible to convince them of their miſtakes. I will, however, offer to them the following demonſtration, which will be underſtood, without difficulty, by every one who knows how to compute (*a*) the increaſe of money at compound intereſt.

The value of a life at 50, (intereſt being at 4 *per cent.*) is 11⅓ years purchaſe by Table VI. For an annuity, therefore, of 44*l.* *per annum* for life, to be enjoyed by a perſon at this age, 498*l.* ought to be given. *Two* in *three* of a number of perſons at the age of 32 will, (by Tables III, IV, and V,) live to 50; and therefore, in order to be able to pay an annuity to them of 44*l.* for life, after 50, the money now advanced by every *three*, ought to be ſuch as will, in conſequence of being laid up to be improved, increaſe in 18 years to double 498*l.* or to 996*l.*—From the preceding Table it may be learnt, that the money which ought to be advanced by every ſingle perſon is 165*l.* or by *three* perſons 495*l.* and this, in 18 years, will double itſelf, or increaſe to juſt the ſum that will then be the value of the annuities to be paid.—But the money required in this caſe by the *Laudable Society*, is 14*l.* 11*s.* 9*d.* from each member at admiſſion, beſides an *annual* payment of 4*l.* The admiſſion-money, therefore, of two members, being 29*l.* 3*s.* 6*d.*

(*a*) The eaſieſt method of doing this, is taught in the rules annexed to the Tables in the APPENDIX.

may be increaſed to twice this ſum, or to 58*l.* 7*s.* An annual payment of 4*l.* for 18 years will, if perfectly improved at 4 *per cent.* compound intereſt, increaſe to 102*l*; and two ſuch annual payments will increaſe to 204*l.*

The whole pay, therefore, of *two* members will produce at the end of 18 years 262*l.* 7*s.* — A third part, I have ſaid, will die without attaining to 50, and theſe will live one with another 10 years. An annuity of 4*l.*, for this time, will produce a capital of 48*l.* and this capital improved for eight years more will increaſe to 66*l.* The whole profit, therefore, from the member who will die is, his admiſſion-money doubled and added to 66*l.* or 95*l.* 3*s.* 6*d.* And this ſum added to 262*l.* 7*s.* makes 357*l.* 10*s.* 6*d.* the *whole* money with which the ſociety can be provided, at the end of 18 years, to bear the expence of *two* life-annities, worth together 996*l.*

By a ſimilar computation it may be found, that the improvement of money at only 3 *per cent.* will *ſink* the *former* ſum to 329*l.* at the ſame time that the value of the *annuities* will be *raiſed* to 1100*l.*

The deficiencies in the ſchemes of all the other ſocieties, except the *Provident Society*, are no leſs conſiderable (*a*). — What confuſion

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(*a*) Some of theſe ſocieties tell us, that the payments on admiſſion ſhall increaſe, as the number of members increaſes; and they have practiſed on this rule juſt as if the ſion then muſt they produce ſome time or other? How barbarous is it thus to draw money

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the value of an annuity was nothing determinate in itſelf, but depended on the number of perſons who have been purchaſers. But the true deſign may perhaps be, to quicken the public in their applications.

Should any of theſe ſocieties, ſenſible of their miſtakes, reſolve to reform themſelves, they ought to conſider, that this cannot be done by only obliging *future* members to pay the juſt values of the annuities promiſed them. All the *preſent* members muſt likewiſe, beſides raiſing their payments, make compenſation for what they have hitherto paid too little; and this compenſation is to be calculated in the following manner. — "Find the whole "amount to the preſent time of the payments which have "been made. Subtract this from the whole amount of "the payments which *ſhould* have been made; and the "*remainder* will be the compenſation required."

Example. In the *Laudable Society of Annuitants*, the condition of a title to 44*l.* *per annum* for life, after 50, to a perſon at the age of 40, was, 4 years ago, 34*l.* 17*s.* in admiſſion-money, beſides an annual payment of 6*l.* 14*s.* 'till he attained to 50.—The admiſſion-money will, (reckoning compound intereſt at 3 *per cent.*) amount in four years to 39*l.* 4*s.* and the annual payment to 28*l.* The whole amount, therefore, of the payments of a member admitted 4 years ago, is 67*l.* 4*s.*—But the value of the annuity was 37*l.* 4*s.* in annual payments, beſides 34*l.* 17*s.* in admiſſion-money; and theſe payments, during the 4 years, would have amounted to 195*l.* The difference, therefore, between theſe two amounts, or 127*l.* 16*s.* is the *compenſation* which ſuch member ought to pay; and if he continues a member without paying it, (beſides raiſing his annual contribution to 37*l.* 4*s.*) he muſt either loſe his annuity, or owe it to injuſtice.

I have taken intereſt here at 3 *per cent.* becauſe I think theſe ſocieties cannot reaſonably depend on always improving the money they receive at a higher rate.

Since I writ the above, I have found, that the admiſſion-money required by this ſociety has lately received

another

ney from the public by promiſes of advantages that *cannot* be obtained? Have we not already ſuffered too much by *bubbles*; and, if nothing elſe can check the frenzy that encourages them, ought not the legiſlature to interpoſe its authority?

I do not, however, mean to condemn all inſtitutions of this kind. They may be very uſeful, if the full values are taken, and proper care is uſed in the *improvement* of money. Intereſt, in theſe caſes, ought not to be reckoned higher than 3 *per cent.* and, ſuppoſing money improved at this rate, a perſon, for a ſingle payment of 50*l.* before he is 40, might be entitled to a life-annuity of 10 guineas *after* 55; or, if he chuſes it, to a life-annuity of 17*l.* *after* 60. But if he pays the ſame ſum before he is 34, he might be entitled to a life annuity of 14*l.* *after* 55, or 22*l.* *after* 60. 25*l.* might purchaſe for him *half* theſe annuities; and 100*l.* *double*.

A ſociety or office that would go on this plan, might do great ſervice. Perſons in

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another advance. At the age of 40, in particular, it is advanced to 108*l.* 7*s.*—when they have further either advanced the admiſſion-money to *double* this ſum, or *tripled* the annual payments, they will be *almoſt* right with reſpect to this particular age, provided the *compenſation-money*, juſt mentioned, has been paid.

Theſe ſocieties, tho' their plans are ſo inſufficient, may, after beginning their payments to annuitants, continue them 15, or, perhaps, 20 years; but it will be by robbing all the younger members.

the lower ſtations of life might be brought to a habit of induſtry, in the beginning of life, by ſtriving to get 25*l.* or 50*l.* beforehand in order to purchaſe ſuch annuities, and thus to make proviſions for themſelves in the more advanced parts of life, when they will be incapable of labour.

There are now eſtabliſhed in *Holland* ſome inſtitutions of this kind.—Any poor perſons there, I am informed, who can, before they attain to a particular age, lay up 50*l.* may make uſe of it in buying for themſelves a right to be admitted, when 50, or at any time afterwards, to houſes prepared on purpoſe, for providing them with all the conveniencies of lodging and board. This is an excellent inſtitution; and I wiſh there was ſome imitation of it in this kingdom.

Conſiderable profits would, in this, caſe be received, from the payments of *ſome* who would chuſe to *delay* going into ſuch houſes; and of others who would grow rich enough to be above them.

It is proper to obſerve here, that inſtitutions of this kind would furniſh one of the *ſafeſt* ways of providing for widows.—A married man might, by paying 100*l.* before his wife attained to 40, entitle her, after 55, or 60, to a life annuity of 21*l.* or 34*l.* Or, by paying the ſame ſum before ſhe attained to 34, he might entitle her, after the ſame ages, to a life annuity of 28*l.* or 44*l.* (*a*); and in this caſe he would have a chance of ſharing himſelf in the benefit of the annuity.

I have called this the *ſafeſt* way of providing for widows, becauſe attended with none of the dangers ariſing from diſproportion of age between men and their wives, and from the admiſſion of perſons labouring under concealed diſtempers.

I cannot conclude this Section, without mentioning the following plan of a proviſion for Old Age.

Let 13 guineas be given as *entrance-money*; and let beſides 1*l.* 2*l.* 3*l.* 4*l.* &c. be given at the beginning of the 1ſt, 2d, 3d, 4th, &c. years, as the payments for theſe years reſpectively; and let the laſt payment be 16*l.* at the beginning of the 16th year. All theſe payments put together will, according to the probabilities of life in the 3d, 4th and 5th Tables, (intereſt being at 4 *per cent.*) entitle a perſon, whoſe age was 40 when he begun them, to an annuity, after 15 years, beginning with 15*l.* and increaſing at the rate of 1*l.* every year, 'till, at the end of 15 years more, or (*b*) when he has attained to 70, it

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(*a*) The ſame payment before 30, would entitle to an annuity of 22*l.* after 50.

(*b*) According to the probabilities of life in the *London* Table, this annuity ſhould be greater.—A *Theorem* for finding what the annuity ought to be in theſe caſes, is given in the Appendix, Note (I).

becomes a ſtanding annuity of 30*l.* for the remainder of his life.

If the addition of three guineas is made to the *entrance-money*, for every year that any life between 30 and 40 falls ſhort of 40, the value will be obtained nearly, of the ſame annuity to be enjoyed by that life, after the ſame number of years, and increaſing in the ſame manner, 'till, in 30 years, it becomes *ſtationary* and *double*.—This plan is particularly inviting, as it makes the *largeſt* payments become due, when the *near* approach of the annuity renders the encouragement to them *greateſt*; and as, likewiſe, the annuity is to increaſe continually with age, till it comes to be higheſt (*a*), when life is moſt in the decline,

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(*a*) The lower part of mankind are objects of particular compaſſion, when rendered incapable, by accident, ſickneſs, or age, of earning their ſubſiſtence. This has given riſe to many very uſeful ſocieties among them, for granting relief to one another, out of little funds ſupplied by *weekly* contributions. A ſociety of this kind, formed on the following plan, would probably thrive, and might, on ſome accounts, be even more uſeful than the inſtitutions in *Holland*, mentioned in p. 118.

Let the ſociety, at its firſt eſtabliſhment, conſiſt of 100 perſons, all between 30 and 40; and whoſe mean age may therefore be reckoned 36; and let it be ſuppoſed to be always kept up to this number, by the admiſſion of new members, between the ages of 30 and 40, as old members die off. Let the contribution of each member be four-pence *per* week, making, from the whole body, an annual contribution of 85*l.* 17*s.*—Let it be further ſuppoſed, that ſeven of them will fall every year into diſorders, that ſhall incapacitate them for ſeven weeks.—

cline, and when therefore it will be moſt uſeful. — It is further a recommendation of this plan, that leſs depends in it on the *improvement* of money than in moſt other plans. — But I muſt leave theſe hints to be purſued by others.

S E C T. V.

Of the Amicable Society for a perpetual Aſſurance Office: And the Society for equitable Aſſurances on Lives and Survivorſhips.

THE 10th Problem has been given, with a particular view to the corporation of the *Amicable Society*, for a perpetual Aſſu-rance-

30*l.* 12*s.* of the annual contribution will be juſt ſufficient, to enable the ſociety to grant to each of theſe 12*s.* *per* week, during their illneſſes. And the remaining 55*l.* *per annum*, laid up and *carefully* improved, at 3½ *per cent.* will increaſe to a capital that ſhall be ſufficient, according to the chances of life in Tables III, IV, and V, to enable the ſociety to pay to every member, *after* attaining to 67 years of age, or *upon* entering his 68th year, an annuity, beginning with 5*l.* and increaſing at the rate of 1*l.* every year for ſeven years, 'till, at the age of 75, it came to be a ſtanding annuity of 12*l.* for the remainder of life.

Were ſuch a ſociety to make its contribution *ſevenpence per* week, an allowance of 15*s.* might be made, on the ſame ſuppoſitions, to every member during ſickneſs; beſides the payment of an annuity beginning with 5*l.* when a member entered his 64th year, and increaſing for 15 years, 'till, at 79, it became fixed for the remainder of life at 20*l.*

If the probabilities of life are lower among the labouring poor, than among the generality of mankind, this plan will be ſo much the more ſure of ſucceeding.

rance-office on ſingle lives, kept in *Serjeant's-Inn*. This ſociety was eſtabliſhed in 1706, and is the only one I am acquainted with, which has ſtood any conſiderable trial from time and experience. The annual payment of each member uſed to be 6*l.* 4*s.* payable quarterly; but it has been lately reduced to 5*l.* The whole annual income, hence ariſing, is equally divided among the *nominees*, or heirs of ſuch members as die every year; and this renders the dividends among the *nominees* in *different* years, more or leſs, according to the number of members who have happened to die in thoſe years. But the ſociety now engages, that the dividends ſhall not be *leſs* than 150*l.* to each claimant, tho' they may be *more.* — None are admitted whoſe ages are *greater* than 45, or *leſs* than 12; nor is there any difference of contribution allowed on account of difference of age.

This ſociety has, I doubt not, been very uſeful to the public; and its plan is ſuch, that it cannot well fail to *continue* to be ſo. It might, however, certainly have been much more uſeful, had it gone from the firſt on a different plan. It is obvious, that regulating the dividends among the *nominees* by the number of members who die every year, is not *equitable*; becauſe it makes the benefit which a member is to receive to depend, not on the value of his contribution, but on a *contingency*; that is, the number of members that ſhall happen to die the ſame year with him. This regulation muſt alſo have been diſadvantageous to the ſociety; as will appear from the following account of the natural progreſs of the affairs of ſuch a ſociety, when eſtabliſhed on a right plan.

Suppoſe a *thouſand* perſons, whoſe common age is 36, to form themſelves into a ſociety for the purpoſe of *aſſuring* a particular ſum at their deaths, to ſuch perſons as they ſhall name, in conſideration of a particular annual-contribution to be continued during their lives. Suppoſe the annual contribution to be 5*l.* and the firſt payment (*a*) to be made immediately. Suppoſe, likewiſe, the original number of the ſociety to be conſtantly kept up by the admiſſion of new members, at 36 years of age, in the room of ſuch as die.—In Queſt. X. p. 33, it appears, that an annual payment, beginning immediately, of 5*l.* during a life at the age of 36, ſhould entitle, at the failure of ſuch a life, to 172*l.* reckoning intereſt at 4 *per cent.* and taking Mr. *De Moivre*'s valuation of lives.—A *thouſand* perſons, all 36 years of age, will die off at the rate of 20 every year. The diſburſements, therefore, of ſuch a ſociety will be, the firſt year, 20 times 172*l.* or 3,440*l.* and its income will be 5000*l.* It will, therefore, at the end of the year, have a ſurplus

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(*a*) Such payments, it has been ſhewn, Queſt. VIII. p. 28, are better than any *half* yearly or *quarterly* payments, and at the ſame time they ſave ſome trouble.

of 1560*l.* to put to intereſt.—In conſequence of the yearly acceſſions to ſupply vacancies, the number dying annually will be always increaſing after the firſt year. In 50 years it will attain to a *maximum*; and then, the affairs of the ſociety will become *ſtationary*, and the number dying annually will be 40, and its annual expence will be 6,880*l.* exceeding the annual contribution 1,880*l.* But, in the mean time, by improving its ſurplus moneys, it will have raiſed a capital equal to this exceſs, and, conſequently, its affairs will be fixed on a firm baſis for all ſubſequent times.

Suppoſe now, that ſuch a ſociety, at its eſtabliſhment, ſhould reſolve to divide its whole yearly income among the *nominees* of deceaſed members. The effect of this would be, that no capital could be raiſed; that the dividends payable to *nominees* would diminiſh continually, till, at the time that the greateſt number of members came to die *annually*, or at the end of 50 years, they would be reduced to half; and all claimants, after this period, receive too little, becauſe the firſt claimants had received too much (*a*).

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(*a*) The reverſe of this will take place, if ſuch a ſociety *begins* with admitting all at all ages, and afterwards changes its plan, and *limits* the age of admiſſion. In this caſe, the number of *yearly deaths* will be *greateſt* at firſt, and the *dividends ſmalleſt*. In conſequence of altering its plan, the *yearly deaths* will leſſen gradually, and the *dividends* riſe; but in time *both* would return again to their original ſtate.

The

At the time of the inſtitution of the *Amicable Corporation,* the intereſt of money was at 6 *per cent.* and, as they admit all between 12 and 45, the mean age of admiſſion cannot probably be ſo great as 36. It appears, therefore, that had they avoided the error now mentioned, and gone from the firſt on the plan I have deſcribed; they might have all along paid to each *nominee* 172*l.* beſides raiſing a capital much greater, in proportion to the number of members, than that I have ſpecified; by the help of the exceſs of their annual payments above 5*l.* and ſome other advantages which they have enjoyed (*a*). Indeed, I cannot doubt but that, with theſe advantages, they might, before this time, have found themſelves able to pay at leaſt 200*l.* to each *nominee*; and at the ſame time

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The following facts incline me to ſuſpect, that this remark may be applicable to the *Amicable Corporation.*

Firſt. In their *original charter*, as it is given in their printed abſtracts, there is no limitation of age mentioned; but 31 years afterwards, I find a bye-law made againſt admitting any perſon who ſhould be above the age of 45, or under 12.—Secondly. In their printed advertiſements in 1770, it is ſaid, that in 59 years they had paid, among 3643 claimants, 378,184*l.* from whence it follows, that tho' the average of their dividends, for the laſt 17 years, has been 154*l.* the ſame average, for 59 years, is only 104*l.*

(*a*) A ſurplus from a *thouſand* members of only *five ſhillings per annum*, duly improved, at 4 *per cent.* would, in 41 years, produce a capital of 25,000*l.*

reſtricted themſelves, as they now do, to an annual payment of 5 *l.* (*a*).

I have already mentioned one inſtance in which the plan of this ſociety is not equitable. Another inſtance of this is, their requiring the ſame payments from all perſons under 45, without regarding the differences of their ages; whereas, the annual payments of a perſon admitted at 45, ought to be double the annual payment of a perſon admitted at 12.

Further. The plan of this ſociety is ſo narrow, as to confine its uſefulneſs too much. It can be of no ſervice to any perſon whoſe age exceeds 45. It is, likewiſe, far from being properly adapted to the circumſtances of perſons, who want to make aſſurances on their lives, for only ſhort terms of years.—Thus; the true value of the aſſurance of 150 *l.* for 10 years, on the life of a perſon whoſe age is 30, is, by Queſt. XIV, (intereſt being at 3 *per cent.*) 2 *l.* 13 *s.* in annual payments for 10 years, to begin at the end of the firſt year; and ſubject to failure when the life fails. But ſuch an aſſurance could not be made, in this ſociety, without an annual payment of 5 *l.*—Neither is the plan of this ſociety at all

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(*a*) It ſhould be remembered, that all this is ſaid on the ſuppoſition, that proper care has been taken to keep out unhealthy perſons; and that the probabilities of life among the members of this ſociety, are the ſame with thoſe in the 3d, 4th, and 5th Tables, in the *Appendix*.

adapted to the circumſtances of perſons, who want to make aſſurances on particular ſurvivorſhips. — For example. A perſon poſſeſſed of an eſtate, or ſalary, which muſt be loſt with his life, has a perſon dependent upon him, for whom he deſires to ſecure a ſum of money, payable at his death. But, he deſires this only as a proviſion againſt the danger of his dying *firſt*, and leaving a wife, or a parent, without ſupport. In theſe circumſtances, he enters himſelf into this ſociety; and by an annual payment of 5*l.* entitles his *nominee* to 150*l.* In a few years, perhaps, his *nominee* happens to die; and, having then loſt the benefit he had in view, he determines to forfeit his former payments, and to withdraw from the ſociety. In this way, probably, this ſociety muſt have gained ſome advantages. But the right method would have been, to have taken from ſuch a perſon the true value of the ſum aſſured, "on the ſuppoſition of non-payment, provided he ſhould ſurvive." In this way he would have choſen to contract with the ſociety; and had he done this, he would have paid for the *aſſurance*, (ſuppoſing intereſt at 3 *per cent.* his age 30, the age of his *nominee* 30, and the probabilities of life as in the 3d, 4th, and 5th Tables) 3*l.* 8*s.* (*a*) in annual payments, to begin immediately, and to

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(*a*) The value of 150*l.* payable at the death of a perſon, aged 30, *provided* he ſurvives another perſon of the ſame to be continued during the *joint* continuance of his own life, and the life of his *nominee*.

All theſe objections are removed by the plan of the ſociety kept in *Nicholas-Lane, Lombard-Street*, which has juſtly ſtiled itſelf the Society for *Equitable Aſſurances on Lives and Survivorſhips*. This Society, if due care is taken, may prove a very great public benefit. It was founded, in conſequence of propoſals which had been made, and lectures, recommending ſuch a deſign, which had been read by Mr. *Dodſon*, the author of the *Mathematical Repoſitory*. It aſſures any ſums, or reverſionary annuities, on any lives, for any number of years, as well as for the whole continuance of the lives, at rates ſettled by particular calculation, and in any manner that may be beſt adapted to the views of the perſons aſſured. That is; either by making the aſſured ſums payable *certainly* at the failure of any given lives; or on *condition* of ſurvivorſhip; and alſo, either by taking the price of the aſſurance in *one preſent payment*; or in *annual payments*, during any ſingle or

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ſame age, is, by Queſt. XI. Chap. I. *l.*45.65; and this value divided by 13.43, (the value increaſed by unity, of two joint lives both 30) gives *l.*3.4, or 3*l.* 8*s.* — The value of the ſame reverſion, according to the probabilities of life in *London*, is, *l.*49.19, in *one* payment; and 4.16, in *annual* payments, during the joint lives, the firſt payment to be made immediately.

joint lives, or any terms leſs than the whole continuance of the lives.—In ſhort; the plan of this ſociety is ſo extenſive, and ſo important, that I cannot ſatisfy my own mind, without offering to the gentlemen concerned in the direction of it, the following obſervations, hoping they will not think them impertinent or improper.

Firſt. They ſhould conſider what diſtreſs would ariſe from the failure of ſuch a ſcheme in any future time; and what dangers there are, which ought to be carefully guarded againſt in order to ſecure ſucceſs. I have already more than once obſerved, that thoſe perſons will be moſt for flying to theſe eſtabliſhments, who have feeble conſtitutions, or are ſubject to diſtempers, which they know render their lives particularly precarious; and it is to be feared, that no caution will be ſufficient to prevent all danger from hence.

Again. In matters of chance, it is impoſſible to ſay, that an unfavourable run of events will not come, which may hurt the beſt contrived ſcheme. The calculations only determine probabilities; and, agreeably to theſe, it may be depended on, that events will happen on the whole. But at particular periods, and in particular inſtances, great deviations will often happen; and theſe deviations, at the commencement of a ſcheme, muſt prove either very favourable, or very unfavourable.

But further. The calculations ſuppoſe, that all the monies received are put out immediately to accumulate at compound intereſt. They make no allowance for loſſes, or for any of the expences attending management. On theſe accounts, the payments to a ſociety of this kind, ought to be ſomewhat more than the calculations will warrant. The intereſt of money ought to be reckoned low; and ſuch Tables of Obſervations uſed as give the higheſt values. Mr. *Dodſon*, I find, has paid due attention to all this, by reckoning intereſt, in his calculations for this ſociety, at 3 *per cent.* and taking the loweſt of all the known probabilities of life, or thoſe deduced from the *London* bills of mortality (*a*). There is, beſides, a liberty provided of making a call on all the members, in caſe of any particular emergency. It is, therefore, highly probable, that this ſociety is ſecure. The laſt expedient, however, would be a very diſagreeable one, ſhould there be ever any oc-

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(*a*) It ought, however, to be remembered here, that in ſelling life-annuities to commence either immediately, or after given terms; and alſo in ſome other caſes, the values come out *leſs* in conſequence of *lower* probabilities of life. Would it, in *ſuch* inſtances, be taking an unfair advantage, to eſtimate the values by the 3d, 4th, or 5th Table in the Appendix, rather than the *London* Table?— Thus; was the ſociety to ſell 20*l.* *per annum*, for life, to a perſon now 30, after attaining to 50, the value, according to Dr. *Halley*'s Table, would, reckoning intereſt at 3 *per cent.* be 90*l.* in a ſingle payment; but, according to the *London* Table, the value would be only 70*l.*

caſion for having recourſe to it; and, in order to guard ſtill more effectually againſt danger, it would not, I think, be amiſs to charge a profit of 3 or 4 *per cent.* on all the payments. —Should the conſequence of this prove, that in ſome future period the ſociety ſhall find itſelf poſſeſſed of too large a capital, the harm will be trifling, and future members will reap the advantage. But this leads me to repeat an obſervation of particular conſequence.

As this ſociety is guided in every inſtance, by ſtrict calculation, it is not to be expected that it can meet with any difficulties for many years; becauſe, not till the end of many years after it has acquired its *maximum* of members, will the *maximum* of yearly claimants and annuitants come upon it? Should it, therefore, thro' inattention to this remark, and the encouragement ariſing from the poſſeſſion of a large ſurplus, be led to check or ſtop the increaſe of its ſtock by enlarging its dividends too ſoon, the conſequences might prove pernicious.

Again; I would obſerve, that it is of great importance to the ſafety of ſuch a ſociety, that its affairs ſhould be under the inſpection of able mathematicians. Melancholy experience ſhews, that none but mathematicians are qualified for forming and conducting ſchemes of this kind.—In ſhort; dangerous miſtakes may ſometimes be committed if the affairs of ſuch a ſociety are not managed carefully and prudently. One inſtance of this I cannot avoid mentioning.

A perſon, who deſires to aſſure a particular ſum, to be paid at the failure of his life, on condition of the ſurvivorſhip of another life, may chuſe to pay the value in annual contributions during the continuance of his own ſingle life, rather than during the continuance of the joint lives, becauſe the annual contributions, in this caſe, ought to be much leſs. But a ſociety that would practiſe ſuch a method of *aſſurance* would hurt itſelf; for, as ſoon as the life, on whoſe ſurvivorſhip the aſſurance depends, is extinct, the perſon aſſured, if then living, would have no longer any benefit in view; and, therefore, would make his payments with reluctance, and, in time, perhaps, entirely withdraw them; the conſequence of which would be, that the ſociety would ſuffer a loſs by being deprived of the juſt value of the expectation it had granted. The plan of a ſociety ought always to be ſuch, as that the loſſes ariſing from diſcontinuance of payment, ſhould fall on the purchaſer, and never on the ſociety.

I muſt not forget to add, that it is neceſſary, that ſuch a ſociety ſhould be furniſhed with as complete a ſet of Tables as poſſible. This will render the buſineſs of the ſociety much more eaſy, and alſo much more capable of being conducted by perſons unſkilled in mathematics. It will alſo contribute much to its *ſafety*. For in all caſes to which Tables can be extended, there would be no occaſion for employing any calculators; and, conſequently, a danger would be prevented to which, tho' it is not *now*, it may *hereafter* be expoſed; I mean, the danger of happening to truſt unſkilful, or careleſs calculators.—Mr. *Dodſon*, I find, has furniſhed this ſociety with ſome important Tables; and his ſkill was ſuch, that there is no reaſon to doubt, but they may be depended on. They have alſo others which, I believe, are ſafe and accurate. But there are ſome ſtill wanting which ſhould be ſupplied; and all ſhould be ſubjected to the examination of the beſt judges, and afterwards publiſhed; together with a minute account of the principles aſſumed, and the method taken in compoſing them. Such a publication would be a valuable addition to this part of ſcience; and it would alſo be the means of increaſing and eſtabliſhing the credit of the ſociety.

In Queſtions 4th, 6th, 10th, 11th, 14th, 15th, and 16th, I have, with a particular view to this ſociety, given rules, by which may be formed every Table it can want, for ſhewing the values of aſſurances on the *whole duration*, or any *terms*, of any *one* or *two* lives, in all poſſible caſes; and nothing but care and attention can be neceſſary to enable any good arithmetician to calculate from them.

Perhaps, this may be as much buſineſs as any one ſociety ſhould undertake. Rules, however, for finding the values of *aſſurances*, in moſt caſes, where the whole duration of any *three* lives is concerned, may be found in Mr. *Simpſon*'s Select Exercifes, from pag. 299 to p. 307; and it is not poſſible they ſhould follow a better guide.

CHAP. III.

Of PUBLIC CREDIT, *and the* NATIONAL DEBT.

THE *National Debt* is a ſubject in which the public is deeply intereſted. Some obſervations have occurred to me upon it, which I think important; and for this reaſon, though foreign to my chief purpoſe in this work, I cannot help here begging leave to offer them to the reader's attention.

The practice of raiſing the neceſſary ſupplies for every public ſervice, by borrowing money on intereſt, to be continued 'till the principal is diſcharged, muſt be in the higheſt degree detrimental to a kingdom, unleſs a plan is ſettled, for putting its debts into a regular and certain courſe of payment. When this is not done, a kingdom, by ſuch a practice, obliges itſelf to return for every ſum it borrows infinitely greater ſums; and, for the ſake of a preſent advantage, ſubjects itſelf to a burden which muſt be always growing heavier and heavier, 'till it becomes inſupportable.

This ſeems to be now the very ſtate of this nation. At the REVOLUTION, an æra in other reſpects truly glorious, the practice I have mentioned begun. Ever ſince, the public debt has been increaſing faſt, and every new war has added much more to it, than was taken from it, during the preceding period of peace. In the year 1700, it was 16 millions. In 1715, it was 55 millions. A peace, which continued 'till 1740, ſunk it to 47 millions; but the ſucceeding war increaſed it to 78 millions; and the next peace ſunk it no lower than 72 millions. In the *laſt* war it roſe to 148 millions; and, at a few millions leſs than this ſum it now ſtands, and probably will ſtand, 'till another war raiſes it perhaps to 200 millions.—One cannot reflect on this without terror.—No reſources can be ſufficient to ſupport a kingdom long in ſuch a courſe. 'Tis obvious, that the conſequence of accumulating debts ſo rapidly; and of mortgaging poſterity, and funding for eternity, in order to pay the intereſt of them; muſt in the end prove deſtructive. Rather than go on in this way, it is abſolutely neceſſary, that no money ſhould be borrowed, except on annuities, which are to terminate within a given period. Were this practiſed, there would be a LIMIT beyond which the national debts could not increaſe; and time would do that *neceſſarily* for the public, which, if truſted to the oeconomy of the conductors of its affairs, might poſſibly never be done.

This, therefore, is one of the propoſals to which, on this occaſion, I wiſh I could engage attention.—I am ſenſible, indeed, that the *preſent* burdens of the ſtate would, in this caſe, be increaſed, in conſequence of the greater preſent intereſt, that would be neceſſary to be given for money. But I do not conſider this as an objection of any weight. For let the annuity be an annuity for a 100 years. Such an annuity is, to the preſent views of men, nearly the ſame with an annuity for ever; and it is alſo nearly the ſame in calculation, its value at 4 *per cent.* being 24½ years purchaſe, and therefore only half a year's purchaſe leſs than the value of a *perpetuity*. Suppoſing, therefore, the public able to borrow money at 4 *per cent.* on annuities for ever, it ought not to give above 1 *s.* 7 *d.* *per cent.* more for money borrowed on annuities for 100 years: But ſhould it be obliged to give a *quarter*, or even an *half per cent.* more (*a*), the additional burdens derived from hence, would

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(*a*) Theſe annuities might be kept 18 years without being much diminiſhed in value; for, ſuppoſing intereſt at 4 *per cent.* an annuity for 82 years, is within a 49th part, or 2 *l.* in 98 *l.* worth as much as an annuity for a 100 years.

Perhaps, in this way of raiſing money, it might be beſt to offer a higher intereſt at firſt, which ſhould fall to a lower, at the end of given intervals. Thus, tho' 4½ for 100 years is equal in value to 5 *per cent.* for 17 years, and after that 4 *per cent.* for 83 years, yet the latter might appear more inviting.

would not be ſuch as could be very ſenſibly felt; and the advantages, ariſing from the neceſſary annihilation of the public debts by time, would abundantly overbalance them.

Theſe advantages would be, indeed, unſpeakably great. By ſuch a method of raiſing money, the expence of one war would, in time, come to be always diſcharged, before a new war commenced; and it would be impoſſible, that a ſtate ſhould ever have upon it, at any one time, the expence of many wars; or any larger debts than could be contracted, within the limited period of the annuities: and, conſequently, it would enjoy the invaluable privilege of being rendered, in ſome degree, independent of the management of its finances by ignorant or unfaithful ſervants.

I muſt add, that it is by no means neceſſary, that the limited period of the annuities ſhould be ſo long as I have mentioned, or 100 years: And that, at any time before the expiration of this period, the public might employ any ſurplus monies, in extinguiſhing part of the annuities, by purchaſing them for itſelf at the market price; and thus it might aid the operations of time, and keep its debts within any bounds, that its intereſt rendered neceſſary.—Our government has, I know, in ſome inſtances adopted the plan now propoſed; but it is to be wiſhed that, inſtead of retracting (*a*) it, as was once done, it had been carried much further.

I am, however, far from intending to recommend this plan as the beſt a ſtate can purſue. There is another method of gaining the ſame end, which is, on many accounts, preferable to it. I mean, "by providing an "annual ſaving, to be applied invariably, "together with the intereſt of all the ſums "redeemed by it, to the purpoſe of diſcharg-"ing the public debts: Or, in other words, "by the eſtabliſhment of a permanent SINK-"ING FUND."

It is well known, that this plan has been alſo adopted by our government; but, tho' capable of producing the *greateſt* effects in the *eaſieſt* and *ſureſt* manner, it has never been carried into execution. It will abundantly appear from what follows that this obſervation is juſt.

Suppoſe the annual ſaving to be 100,000*l.* This ſum, applied now to diſcharge an equal debt, bearing intereſt at 4 *per cent.* will tranſfer to the public, from its creditors, an annuity of 4,000*l.* At the end of a year, then, there would be a ſaving of 104,000*l.* which would transfer to the public another annuity of 4,160*l.* and make the ſaving, at the

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(*a*) In the year 1720, the nation was put to the expence of three millions, in order to reduce ſeveral long and ſhort annuities then ſubſiſting, to redeemable *perpetuities*.

the end of two years, to be 108,160*l.*— Thus, the original fund would go on increaſing, at the ſame rate with money improved at 4 *per cent.* compound intereſt.—At the end of three years it would be 112,486*l.* At the end of 18 years, 202,587*l.* Of 36 years, 410,393*l.* and of 95 years (*a*), 4,151,138*l.*— At the end of 93 years, then, the nation might be eaſed of above 4 millions *per annum* in taxes; and above 100 millions of its debts would be diſcharged, gradually and inſenſibly, at no greater expence than 100,000*l.* *per annum*; and, without interfering with any of the reſources of government; or making any other difference, than cauſing *funds* to be engaged for a courſe of time to the *public*, that would have been otherwiſe neceſſarily engaged to its *creditors*, and which, therefore, muſt have been entirely uſeleſs to it.

It is an obſervation that deſerves particular attention here, that, on this plan, it is of little importance what intereſt a ſtate is obliged to give for money: For the higher the intereſt, the ſooner will ſuch a fund pay off the principal. Thus; a 100 millions borrowed at 8 *per cent.* and bearing an annual intereſt of eight millions, would be paid off by a fund, producing annually 100,000*l.* in 56 years; that is, in 39 years leſs time, than if the ſame money had been borrowed at 4 *per cent.*

(*a*) See the Queſtions annexed to the Tables in the Appendix.

It follows from hence, that reductions of intereſt would, on this plan, be no great advantage to a ſtate. They would, indeed, lighten its *preſent* burdens; but this advantage would be balanced, by the addition that would be made to its *future* burdens, in conſequence of the longer time, during which it would be neceſſary to bear them.—I mean this on the ſuppoſition, that the ſavings produced by reductions of intereſt, are immediately applied to the relief of the ſtate, by annihilating taxes equivalent to them. But if this is not the caſe; and if, likewiſe, there is either no plan eſtabliſhed for putting the public debts into a certain courſe of payment, or it is not faithfully carried into execution; in theſe circumſtances, reductions of intereſt may prove hurtful. For, firſt, They would only furniſh with more money for ſupplying the deficiencies ariſing from bad management. And, ſecondly, As, in ſuch circumſtances, they would only *retard*, and not *prevent* the increaſe of the burdens occaſioned by the public debts, a period would come when the affairs of the ſtate would get to a *criſis*; and, at ſuch a period, its danger would be increaſed, in proportion to the reductions of intereſt that had been made.

In order to underſtand this; let us ſuppoſe, that a debt, bearing an annual intereſt of five millions, is the whole debt, which a ſtate can bear without being ſo much oppreſt as to be near ſinking. Let it, however, be ſuppoſed to have ſtill ſome laſt reſources left, which may enable it to bear, for 23 years to come, this load, together with every additional load, which, during this time, may be neceſſary to be thrown upon it.—Let it further be ſuppoſed, that at this time, the ſtate, urged by the fear of an approaching bankruptcy, reſolves upon entering into ſome effectual meaſures for preſerving itſelf.—Certain it is, that in this caſe, no meaſure *ſo* effectual can be purſued, as the eſtabliſhment of a *ſinking fund*, and ſuch a faithful application of it as I have explained. Let this then be the meaſure entered upon; and let the ſtate be ſuppoſed capable of providing a fund, producing a million annually. If all the debts bear intereſt at 6 *per cent.* this fund would pay off three fifths of them, within the time I have mentioned; or, in 23 years; and the ſtate might be ſaved. But if, in conſequence of reductions, they bear intereſt at no more than 3 *per cent.* the ſame fund would not give the ſame relief, in leſs than *double* that time; and, therefore, a bankruptcy might prove unavoidable (*a*).

(*a*) In ſome other kingdoms a ſpunge might be applied in ſuch circumſtances, or the funds reduced one half by an act of deſpotiſm, without occaſioning any convulſions; but this is not poſſible in this free country; and, it is to be hoped, never will be poſſible.

I wiſh I could think, that there is nothing in this repreſentation, that can be applied to the preſent ſtate of this nation. The intereſt of the public debts has been reduced, at different periods, from 6 to 5, and from 5 to 4, and 3 *per cent.*; but ſtill they have grown with rapidity; and we now ſee ourſelves overloaded, and in no way of gaining relief. Had there been no reductions of intereſt, we ſhould, indeed, have been in the ſame condition ſooner; but, we might have been relieved alſo ſooner, and with leſs difficulty and danger.

What I have now ſaid implies, that a ſtate always diſcharges its debts, whatever intereſt they bear, by paying the original ſum borrowed. It may, perhaps, be imagined, that when a loan is under *par*, it may be diſcharged at a leſs expence. But this is by no means ſo practicable as it may ſeem; for it ſhould be conſidered, that a public loan, now under *par*, would not long keep ſo, after being put into a courſe of payment: And, for this reaſon, as a ſtate can never be obliged, in redeeming its debts, to pay *more* than the original ſum borrowed, ſo neither ought it to expect, in general, to be able to redeem them by paying *leſs*. I have ſaid, *in general*; for I am ſenſible, that at the beginning of the operations of a fund, when its produce is ſmall; and alſo, in a time of war, a ſtate might derive great advantages from the low price of its debts. And I am ſenſible alſo, that conſiderable advantages might be derived from *lotteries* (*a*), in paying the public debts: But *lotteries* do great miſchief in a ſtate, by foſtering the deſtructive ſpirit of gaming. It is wretched policy to make them familiar, by recurring to them in the ordinary courſe of government. There are great occaſions on which they may be neceſſary, and for ſuch occaſions they ſhould be reſerved.

But to return to the ſubject I have principally in view.

The advantages of putting the public debts into ſuch a courſe of payment, as I have deſcribed, are ſcarcely to be imagined. It would give a vigour to public credit, which would enable a ſtate always to borrow money eaſily, and on the beſt terms. And the encouragement to lenders might be always improved, without any inconvenience, by making every loan irredeemable, during the firſt 20 or 30 years; for, there could ſeldom be any occaſion, for beginning to diſcharge any *one* loan ſooner.

It might be eaſily ſhewn, that the faithful application, from the beginning of the year 1700,

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(*a*) Thus; 800,000*l.* of the 3 *per cents.* at 87; or 1,000,000, at 70, might be redeemed with half a million of money, conſiſting of 50,000 lottery tickets at 10*l.* each, real value; but capable of being ſold at 14*l.*, as was done in ſome of the laſt lotteries.

1700, of only 200,000*l.* annually, would long before this time, notwithſtanding the reductions of intereſt, and every waſte that has been made of the public money, have cauſed above half the public funds to revert to the public, and paid off above 80 millions of its debts. The nation might therefore, ſome years ago, have been eaſed of the greateſt part of the taxes with which it is loaded. The moſt important relief might have been given to its trade and manufactures; and it might now have been in much better circumſtances, than at the beginning of the laſt war; its credit firm; reſpected by foreign nations; dreaded by its enemies; and ready to puniſh any inſult that could be offered to it. The near view, likewiſe, of ſuch a period, during the courſe of the laſt war, would have given higher ſpirits to the nation, and encouraged it to bear the expence occaſioned by the war with more chearfulneſs, and to continue it with vigour for two or three years longer; the conſequence of which would, probably, have been, gaining a full indemnification from our enemies, and weakening them to ſuch a degree, as would have given us effectual ſecurity againſt them for many years to come.—A new account might alſo now have been begun; and another fund, not much more conſiderable, applied in the ſame way, would, in 60 or 70 years more, have paid, not only all that would have been now unpaid, but alſo, probably, a great proportion of ſuch further debts as muſt be contracted within this time (*a*). And thus, without any expence that could be ſenſibly felt, its debts, as ſoon as they began to grow heavy, might have been conſtantly reduced to a *half*, or a *third*; and not only all *danger*, but all conſiderable *inconvenience* from them prevented.

All I have now ſaid, ſuppoſes a *ſingle* fund with a *general* appropriation to the payment of the public debts. The ſame ends might be anſwered by *particular* funds, with ſmall ſurpluſſes, appropriated to *particular* debts. In the wars of King *William* and Queen *Anne*, 6 *per cent.* intereſt was given for all loans. It would have been eaſy to have annexed to each loan a *fund* producing a *ſurplus* of 1 *l. per cent.* after paying the intereſt; and ſuch a *ſurplus* would have been ſufficient to annihilate the principal of every loan in 33 years. Had this plan been followed, the diſengagement of the public funds, and the relief attending it, would have begun 50 years ago; and the debts contracted, during the reigns of

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(*a*) One of the propereſt objects of taxation in a ſtate is *celibacy*. I doubt not, but that by a fund ſupplied only from hence, the end I have in view might have been eaſily accompliſhed; and, conſequently, the very means of paying off the debts of the nation, rendered at the ſame time the means of increaſing its chief ſtrength, by promoting population in it.

of King *William* and Queen *Anne*, would have been all cancelled near 20 years ago, without any of that trouble, tumult and diſtreſs, which have been occaſioned by reductions of intereſt, and by the various ſchemes which have been tried for leſſening the debts (*a*). — A fund, yielding 1 *l. per cent.* ſurplus, annexed to a loan at 5 *per cent.*, would diſcharge the principal in 37 years (*b*). At 4 *per cent.*, in 41 years. At 3 *per cent.*, in 47 years.

Theſe obſervations relate only to what *might* have been the ſtate of the nation with reſpect to its debts, had a right plan been purſued from the firſt. But it will be aſked, What can be done with them as they *are?*— I wiſh I was able to give a more ſatisfactory anſwer to this enquiry. Every one muſt ſee our proſpect to be diſcouraging, and our ſtate hazardous. Some have thought, that a good

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(*a*) The ſums to be laid out would, in this caſe, be ſo ſmall at firſt, that it would be proper to employ them in purchaſing part of the loan to be annihilated at the prices in the public market; and this, as far as it can be carried, is the moſt eaſy and quiet and ſilent way poſſible of extinguiſhing the public debts.

(*b*) I have all along ſuppoſed the produce of the public funds to come in yearly. The truth is, that it comes in *half*-yearly; but this gives no advantage in the payment of the public debts worth taking into account. 1 *l. per annum*, together with its growing intereſt, at 4 *per cent.* taken *yearly* out of 100*l.* will reduce it to nothing in 41 years; if taken *half*-yearly, it will annihilate the ſame capital only four months and 12 days ſooner. See the queſtions annexed to the Tables in the *Appendix*.

method might be found out of diſcharging the national debt, by life annuities. The following obſervations will ſhew how vain an imagination this is.

Let us ſuppoſe, that 33,333,000*l.* is to be paid off, by offering to the public creditors life-annuities, in lieu of their 3 *per cents.* A life at 60, ſuppoſing intereſt at 3½ *per cent.*, and the probabilities of life as in the *Breſlaw*, *Norwich*, and *Northampton* Tables of Obſervation, is worth 11 years purchaſe. A life at 30 is worth 6½ years purchaſe. Certainly, therefore, no ſcheme of this kind would be ſufficiently inviting, which did not offer 8 *per cent.* at an average, to all ſubſcribers. Let us, however, ſuppoſe, that no more than 7½ is given; and that there are 33333 ſubſcribers, at 1000*l.* ſtock each, for which a life-annuity is to be granted of 75*l.* or, for the whole ſtock ſubſcribed, two millions and a half. A million and a half extraordinary, therefore, muſt be provided towards paying theſe annuities.

Let us further ſuppoſe, that the ſubſcribers are perſons between the ages of 30 and 60; and that the numbers of them, at all the intermediate ages, are in the ſame proportions to one another, with the proportions of the living at theſe ages, as they exiſt in the world, or, as they are given in *Tables of Obſervation*. Let us again ſuppoſe, that as theſe *annuitants* die off, they are immediately replaced by others, who are continually offering themſelves at the ſame ages, and in the ſame proportional numbers at theſe ages, with thoſe of the original ſubſcribers at the time they ſubſcribed; in conſequence of which, the whole number of annuitants will be kept always the ſame. In theſe circumſtances, it will be 30 years, at leaſt, before a number will die off, (*a*) equal to the whole number; that is, before 33 millions of debts will be annihilated. But had the extraordinary million and half provided for paying theſe annuities, been employed during this time, in paying off ſo much of the debt at *par* every year, extinguiſhing at the ſame time every year an equivalent tax, 45 millions would have been paid. But had the ſavings, alſo, inſtead of being ſunk as they aroſe, been employed in the ſame manner, 71 millions would have been paid.

The nation, therefore, muſt, without doubt, loſe greatly by all ſchemes of this kind; and yet they have been often much talked of; and, indeed, I ſhall not wonder, ſhould I hereafter ſee an attempt made to pay off the national debt in this way.

I muſt beg leave to detain the reader here ſome time longer. A more particular explanation of this ſubject, will lead to ſome

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(*a*) A demonſtration of this will be given in the Appendix, note (K).

obſervations on the beſt methods of raiſing money which, I think, deſerve to be carefully conſidered.

When any ſum is ſaid to be the *value* of a life-annuity, the meaning is, that, in conſequence of being improved at intereſt, and allowing for the chances of mortality, it will bear the whole expence of the annuity. If, therefore, inſtead of being *laid* up for improvement, it is either immediately applied to particular uſes, or has been long ſince ſpent; there will be a loſs, equal to the ſum which would have been added to the purchaſe-money, had it been *improved*.—This is the reaſon of the loſs which, I have ſhewn, the public would ſuffer by offering *life-annuities*, in lieu of *ſtock*, in order to extinguiſh its debts. And for the ſame reaſon, it muſt always loſe conſiderably by *raiſing* money on life-annuities.

Suppoſe a million raiſed by *annuities* on a ſet of lives, all at 30 years of age. Perſons at this age have, (according to Tables III, IV, and V,) an *expectation* of 28 years. That is; the duration of their lives, taking them one with another, will be 28 years; (ſee the beginning of the firſt Eſſay) and they will be entitled, ſuppoſing intereſt at 4 *per cent.* to 7*l.* *per annum*, for every 100*l.* advanced. For a million then, the public would make 28 payments of 70,000*l.*—Let us ſuppoſe next, that a fund producing this ſum annually, inſtead of being engaged to pay theſe life-annuities, is engaged for 28 years, to pay the principal and intereſt of a million, borrowed on *redeemable* perpetuities, at 4 *per cent.* There will, at the end of the firſt year, be a ſurplus of 30,000*l.*—In conſequence of applying this to the extinction of the principal, it will be reduced to 970,000*l.* on which, at the end of the ſecond year, the intereſt due will be 38,800*l.* There will, therefore, be a ſaving of 1200*l.* Inſtead of employing this ſaving in further ſinking the *principal*, which would cauſe the fund to accumulate in the ſame manner with money at compound intereſt, let it be taken and employed in any other way: And let the ſame be done with all the ſubſequent ſavings, reſerving only 30,000*l.* annually, for the purpoſe of ſinking the principal. At the end of the ſecond year, the principal will be 940,000*l.*; and the ſaving of intereſt upon it, at the end of the third year, 2400*l.* At the end of the 28th year, the principal will be reduced to 160,000*l.* The ſaving of intereſt that year will be, 1200*l.* multiplied by 27, or 32,400; and the ſum of all the ſavings will be 453,600*l.*—Deduct from hence 160,000*l.* remaining then undiſcharged of the principal; and 293,600*l.* will be the loſs the public would ſuſtain, in the circumſtances I have ſuppoſed, by raiſing money on life-annuities. But if we ſuppoſe the ſavings, as they ariſe, as well as the con-

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ſtant ſum of 30,000*l.* to be applied to the diſcharge of the principal, inſtead of being ſpent on current ſervices; the whole million will be annihilated in 21 years and half; and the loſs to the public by life-annuities, will be $6\frac{1}{2}$ years purchaſe of the annuities; or 455,000*l.*—By ſimilar deductions it may be eaſily found, that the loſs, in *younger* lives, is greater; in *older* lives leſs; but never inconſiderable, except in the *oldeſt* lives.

It appears, therefore, that, in conſequence of ſuch a way of raiſing money, the public muſt always pay much more in intereſt than there is any occaſion for; and *waſte* a ſum nearly equal to half the principal borrowed (*a*). This,

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(*a*) It is obvious, that the obſervations here made, may be applied to the common methods of raiſing money, on life-annuities, for building churches, paving ſtreets, making navigations, &c. &c. And, in general, to all caſes where the money received, is not laid up to be improved. —For, to view this ſubject in another light, let us ſuppoſe 10,000*l.* borrowed for any public work, on perpetuities, at 4 *per cent.* And, if that will afford more encouragement, let them be made irredeemable for any number of years leſs than ſeventeen. Let us further ſuppoſe, ſuch rates, or tolls, eſtabliſhed for the payment of the intereſt and principal, as ſhall produce *double* the intereſt of the ſum borrowed; or 800*l. per annum*, inſtead of 400*l. per annum.* Let the *ſurplus*, as it comes in *half-yearly*, be laid up to accumulate in the public funds. In 17 years and half, reckoning intereſt at 4 *per cent.* a capital will be raiſed, equal to the whole ſum borrowed; and, therefore, at the end of that time, the whole debt may be diſcharged, and the whole tranſaction finiſhed.— But if the ſame ſum had been borrowed on annuities, for he

This, however, tho' ſo waſteful, is a more frugal way of procuring money than by borrowing on *perpetuities*, without putting them into a courſe of redemption; for in this caſe, (if a ſpunge is not applied) the loſs muſt be *infinite*.

I muſt add, that theſe obſervations are particularly applicable to all the ways of raiſing money by the ſale of reverſions.—The public, for inſtance, might procure a million, by offering for it a fund, that will be diſengaged at the end of 18 years; and then produce 80,000*l.* *per annum* for ever. This, ſuppoſing intereſt at 4 *per cent.*, would be the very ſame with offering *two* millions, 18 years hence, for *one* million now: And a private man, or an *office* for the ſale of reverſions, might gain by ſuch a tranſaction; becauſe, the money advanced, in conſequence of being improved, might, in 18 years, be more than doubled.

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the lives of a ſet of perſons 50 years of age, at 8 *per cent.* which is 1 *l. per cent.* leſs than the true value of ſuch annuities: Had this, I ſay, been done, *half* the annuitants would have been alive at the end of the term I have mentioned; (ſee Tables III, IV, and V,) and the whole tranſaction, together with the expences and trouble attending the management of it, could not have been finally cloſed 'till the extinction of all the lives; that is, not in leſs time, moſt probably, than 35, or, perhaps, 40 years. —It is a neceſſary obſervation here, that, if public credit maintains its ground, much will not depend, in the plan now propoſed, on the riſe and fall of STOCKS. If a *war* ſinks them, the money laid out, while the war laſts, will accumulate faſter. If a peace raiſes them, the money that had been previouſly laid out will be proportionably increaſed.

doubled. But, as the *public* always borrows for immediate ſervices, and never lays up money, it would neceſſarily loſe a ſum equal to the whole ſum borrowed: And the ſame money might have been borrowed on a fund, producing 50,000*l. per annum*; which would not only pay the intereſt, but diſcharge the whole principal in 41 years (*a*).

By raiſing money on life-annuities, the *preſent members* of a ſtate take a heavier load on themſelves, in order to exempt *poſterity*; and there would be a laudable generoſity in this, were it not for the *folly* of it; the ſame exemption being equally practicable at *half* the expence.—On the other hand. By borrowing on *reverſionary* grants, the *preſent* members of a ſtate exempt themſelves *entirely*, by throwing the load *doubled* on poſterity; and there is a cruelty and injuſtice in this that nothing can excuſe.

It is well known, that both theſe methods of raiſing money have been practiſed among us. This, however, is, by no means, the worſt that has been done. It has been common to borrow money to pay the intereſt of money borrowed, and thus to give *compound intereſt* for money; and our *parliaments* have, ſometimes, expreſsly provided, that this ſhall be done for a ſucceſſion of years.

(*a*) The ſmallneſs of the ſums, which I have here and elſewhere ſometimes ſuppoſed to be employed in diſcharging the public debts, can create no difficulties, becauſe there is no ſum which may not be applied to this uſe by purchaſing ſtock.

But to return to the main point I had in view.

The enquiry which has occaſioned this digreſſion, muſt be highly intereſting to every perſon who wiſhes well to his country.—All ſchemes for diſcharging the public debts, by life-annuities, have been ſhewn to be abſurd and extravagant.—In general; it may be obſerved, that it is far from probable, that any money which the nation can ſpare, if applied ſo as to bear only *ſimple* intereſt, can be capable of reducing its debts within due bounds; or of doing us, in our preſent circumſtances, any eſſential ſervice. A fund, producing a ſurplus of even two millions annually, would, when thus applied, pay no more than 40 millions in 20 years; and, in that time, a war might probably come, which would interrupt the application of it; and increaſe our debts much more than ſuch a fund had leſſened them.

Certain it is, therefore, that if our affairs are to be retrieved, it muſt be by a *fund* increaſing itſelf in the manner I have explained. The ſmalleſt *fund* of this kind is, indeed, *omnipotent*, if it is allowed time to operate. But we are, I fear, got ſo near to the limits of the reſources of the nation, that it cannot be allowed much time: And, in order to make amends for this, it is neceſſary that it ſhould be *large*.—Let us then ſuppoſe, that the nation is ſtill ſtrong enough to enable it to provide a fund, that ſhall yield a *million* and *half annually*, for 20 years to come: And alſo, that, together with all its *preſent* burdens, it is capable of bearing every *additional* burden that 20 years more can bring upon it. If this is not true, we have, I think, nothing to do but to wait the iſſue, and tremble.

A fund, producing annually a million and a half, would increaſe to three millions *per ann.* in 20 years (*a.*) At the end of this term, the nation might be eaſed of the moſt oppreſſive taxes, to the amount of a million and a half; and the conſequence would prove, that, if there ſhould have been a war, either the whole, or much the greateſt part of the addition occaſioned by it to the public burdens, would be taken off, and the nation reinſtated nearly in its preſent circumſtances. But, if there ſhould have been no war, the national debt and the taxes charged with it, would be reduced a third below the ſums at which they now ſtand; and the nation would be ſo much relieved as to be prepared for a war.—The remaining million and half would,

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(*a*) It ſhould be remembered, that in the year 1779, 1 *l. per cent.* on the conſolidated 4 *per cents*, will be annihilated, and that I ſuppoſe the ſavings derived from hence to be taken at that time as a part of the fund.—Methods might be eaſily contrived for getting this ſaving immediately, which would be ſome advantage.

in 23 years, increaſe again to three millions *per annum*; and then, ſo much more of the public taxes would be ſet free; 50 millions more, or 93 millions in all, of the public debts would be diſcharged, and the difficulties of the nation would be, in a great meaſure, conquered.—During this whole courſe of time, there may poſſibly be but one war; and ſhould that happen, the appropriation at the end of it, of about 400,000*l.* *per annum*, might be enough to anſwer all purpoſes.

In theſe obſervations, I ſuppoſe the 3 *per cents* to be paid off at *par*; and no advantage taken at any time of their low price. By taking this advantage, and with the help of a little management, a fund, producing annually a million and half, might be made to increaſe to another million and half, in leſs time than I have aſſigned. Should there be a war in a few years, the 3 *per cents.* would probably fall below 75; and then the proprietors of them muſt be glad to part with them at this price; the conſequence of which, ſuppoſing the war to laſt eight years, would be, that the fund would double itſelf, and the nation be relieved in the manner I have mentioned, in 18, inſtead of 20 years.—The advantage will be the ſame, ſuppoſing the government at ſuch a time to go on in paying off the 3 *per cents* at *par*. For the effect of this would be, that money might be borrowed for the public ſervice on proportionably better terms. Suppoſe, for inſtance, that four millions muſt be borrowed for the ſervice of the year; and let the produce of the fund be then increaſed to two millions; and the intereſt of money in the *ſtocks,* above 4 *per cent.* In theſe circumſtances, it would be the intereſt of the lenders of money, to take 3½ *per cent.* for the ſums they advanced, in conſideration of having their 3 *per cents* paid off at *par,* to the amount of half theſe ſums.—War, therefore, would accelerate the redemption of the public debts; and it would do this the more, the longer it laſted, and the higher it raiſed the intereſt of money. Or if, in conſequence of paying always at *par,* this could not happen; an equivalent effect would be produced in the way juſt mentioned. The ſtocks would be always kept up by the operations of the fund; and, in proportion to the ſums yielded by it, the public would be able to borrow money more advantageouſly, and leſs would be added to its burdens.—This ſeems to me an obſervation of particular conſequence. It demonſtrates, that the invariable application, in *war* as well as *peace,* of the produce of the fund I am ſuppoſing, to the payment of the national debts, rather than to any current ſervices, would, independently of its effect in (*a*) redeeming theſe debts, be attended

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(*a*) So true is this, that a war, were we now engaged in it, would only render the *preſent* time ſo much the more ed with great advantages to the public. But this is a ſubject on which I ſhall have occaſion to ſay more preſently.

more proper for entering into meaſures for paying the public debts. And the following obſervations will put this out of doubt.

As it is now become the practice to have recourſe to *lotteries* in *peace*, we may be ſure, that no year will paſs without them in *war*. I would, therefore, propoſe, that, inſtead of making uſe of them in raiſing the annual ſupplies in war, they ſhould be then applied as an aid in diſcharging the public debts.—Suppoſe the war to laſt 10 years, and the 3 *per cents* at 70. Suppoſe alſo, each *lottery* to conſiſt of 750,000*l.* in *tickets*, which, when diſpoſed of to ſubſcribers, will bring in 1,050,000*l.* On theſe ſuppoſitions, the whole *loſs* to the public, from applying the lotteries to the payment of the public debts, rather than to the current ſupplies, will be 1,050,000*l.* annually, or 10 millions and ½ in all.—The *gain* will be as follows. 750,000*l.* of the produce of the ſinking fund, formed into tickets, will be the ſame with 1,050,000*l.*; and this ſum will pay off a million and a half of the 3 *per cents*, every year, or 15 millions in all; and the growing ſavings ariſing from theſe payments will, at the end of 10 years, have paid, at leaſt, two millions more. The nation, therefore, having paid off 17 millions of its debts, and added to them only 10 millions and ½, will gain ſix millions and ½. But this will be the ſmalleſt part of its gains. All the produce of the *ſinking fund*, over and above 750,000*l.* might be charged with the payment of the intereſt of ſuch new debts as would be neceſſary to be contracted during the war; and, at the end of it, the nation, with the help of 200,000*l.* to be diſengaged in 1779, by the reduction of the 4 *per cents*, would find itſelf poſſeſſed of a *fund*, producing 1,450,000*l.* annually; which, faithfully employed, might probably be ſufficient to extricate it from all its difficulties.—Beſides this; ſuch a ſcheme would not only *preſerve*, but *raiſe* and *eſtabliſh* the credit of the public: And he only can be duly ſenſible of the importance of this, who will conſider, what danger there would be in another

The *ſinking fund,* in its preſent ſtate, and, after ſupplying the deficiencies of the peace eſtabliſhment, yields, I ſuppoſe, a conſiderable part of the million and a half I have mentioned. An annual lottery might eaſily raiſe 200,000*l.* more. But this is a meaſure which I cannot wiſh to ſee carried into execution, unleſs abſolutely neceſſary. Were the managers of our affairs ſufficiently in earneſt in this buſineſs, I cannot doubt but that ſuch ſavings might be made in the collection and expenditure of the national revenue, as would cauſe the ſinking fund to yield, for 18 or 20 years to come, the *whole* of this ſum, without impoſing any new burdens on the public. But, were there, indeed, no way of providing any part of it, but by creating new funds, or impoſing new taxes; it *ought* to be done, becauſe it *muſt* be done, or the nation be ruined.

The evils and dangers, attending an *exorbitant* public debt in this country, are ſo great, that they cannot be exaggerated.—Without repeating, what has been ſo often ſaid, of its increaſing the dependence on the crown, by jobs and places without number; occaſioning

ther war, ſhould it continue long, of either *overwhelming* public credit; or of being terrified, by the apprehenſion of ſuch a calamity, into an ignominious and fatal peace. The eſtabliſhment, therefore, of ſome ſuch plan as that now propoſed, would, at the beginning of a war, be the moſt important of all works.

execrable practices of the Alley; rendering us tributary to foreigners; and raiſing the price of proviſions and labour; and, conſequently, checking population, and loading our trade and manufactures; I will only take notice of the two following evils which attend it.

In the firſt place. It muſt check the exertions of the ſpirit of liberty in the kingdom. The tendency of every government is to deſpotiſm; and in this it muſt end, if the people are not conſtantly jealous and watchful. Oppoſition, therefore, and reſiſtance are often neceſſary. But they may throw things into confuſion, and occaſion the ruin of the public funds. The apprehenſion of this muſt influence all who have their intereſt connected with the preſervation of the funds, and incline them always to acquieſcence and ſervility.

But further. It expoſes us to particular danger from *foreign* as well as *domeſtic* enemies, by making us fearful of war, and incapable of engaging in it, however neceſſary, without the hazard of bringing on terrible convulſions by overwhelming public credit.

All theſe are evils which muſt increaſe with every increaſe of the national debt; and there is a point at which, when they arrive, the conſequences muſt be fatal (*a*). — I am

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(*a*) "Either the nation (Mr. *Hume* ſays, Eſſays Vol. II. p. 145,) muſt deſtroy public credit; or public "credit will deſtroy the nation."—A dreadful alternative! ſurely.

now writing under a conviction, that I am doing the little in my power to preſerve my country from this danger. I have ſhewn, that an annual ſupply of a million and a half for 18, or at moſt 20 years, may be made the means of reſtoring and ſaving us. This, therefore, is our remedy; and it ought to be applied *immediately*, leaſt it ſhould not be applied time enough.

But to proceed to ſome further obſervations.

What has been ſaid, has all along ſuppoſed a *ſacred* and *inviolable* application of the fund I have deſcribed, and of all its earnings, to the purpoſe of ſinking the national debt. The whole effect of it depends on its being allowed to operate, WITHOUT INTERRUPTION, a proper time. But it may be aſked, how this can be ſecured? Or, by what method an object, that muſt be continually growing more and more tempting, can be defended againſt invaſion and rapine? — I might here mention the ſuperintendency and care of the repreſentatives of the kingdom, the faithful guardians of the ſtate, to whom miniſters are reſponſible for the uſe they make of the public money. But experience has ſhewn, that we cannot rely on this ſecurity.—The difficulty, therefore, now mentioned, is the very greateſt difficulty the nation has to ſtruggle with in the payment of its debts.

The ſinking fund was eſtabliſhed in the year 1716, or ſoon after the acceſſion of the preſent family, at a time when the public debts, tho' not much more than a third of what they are now, were thought to be ſo conſiderable as to be alarming and dangerous. It was intended as a SACRED DEPOSIT never to be touched; the law which eſtabliſhed it declaring, that it was to be applied to the payment of the principal and intereſt of ſuch national debts and incumbrances, as had been incurred before the 25th of *December* 1716; *and to no other uſe, intent or purpoſe whatever.* — The faith of *parliament*, therefore, as well as the ſecurity of the kingdom, ſeemed to require, that it ſhould be preſerved carefully and rigorouſly from alienation. But, notwithſtanding this, it has been *generally* alienated; and the produce of it employed, in helping to defray ſuch current expences as the exigencies of the ſtate rendered neceſſary.

In order to juſtify this, it has been uſual to plead, that when money is wanted, it makes no difference, whether it is taken from hence, or procured by making a new loan. There cannot be a worſe ſophiſm than this. The difference between theſe two methods of procuring money is no leſs than *infinite.*—For, let us ſuppoſe, a *million* wanted for any public ſervice. If it is borrowed at

4 *per cent.* the public will loſe by the payment of intereſt 40,000*l.* the firſt year, and the ſame the ſecond year, and the ſame for ever afterwards. But if it is taken out of the *ſinking fund*, the public will loſe 40,000*l.* the *firſt* year; 4160*l.* the *ſecond* year; 80,000*l.* the 18th year; a *million* the 85th year: For theſe are the ſums that would, at theſe times, have otherwiſe neceſſarily reverted to the public. It loſes, therefore, the advantage of paying in 85 years, with money of which otherwiſe no uſe could have been made, *twenty-five* millions of debt.—In other words; by employing the SINKING FUND, in bearing current expences, rather than borrowing *new* money; the ſtate, in order to avoid giving *ſimple intereſt* for money, is made to alienate money, that *muſt* have otherwiſe been improved at *compound intereſt*; and that, in time, would have *neceſſarily* increaſed to *any* ſum (*a*).--Had a faithful uſe been made from the firſt, of only one THIRD of the produce of this fund, near *three fourths* of our preſent debts might now have been diſcharged; and, in a few years more, the *whole* of them might have been diſcharged (*b*).—Can it be poſſible then to

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(*a*) The principal obſervations in this Chapter, I have given juſt as they occurred to my thoughts, without knowing that any of them had been made by other writers. Some propoſals of a ſimilar nature, but very differently repreſented, I have ſince found in Mr. *Poſtlethwayt's* Dictionary, under the articles *Public Credit*, *Debts*, *Funds*, &c.

(*b*) See a particular explanation and proof of this in the Queſtions following the Tables in the *Appendix.*

to think, without regret and indignation, of that miſapplication of this fund, which, with the conſent of parliaments always complying, our miniſters have practiſed?—I find it difficult here to ſpeak with calmneſs.—But I muſt reſtrain myſelf. *Calculation*, and not *cenſure*, is my buſineſs in this work.—I muſt believe, that the grievance I have mentioned, has proceeded more from inattention and miſtake, than from any deſign to injure the public.

to think, without regret and indignation, of that misapplication of this fund, which, with the consent of parliaments always complying, our ministers have practised ? — I find it difficult here to speak with moderation. — But I must refrain [illegible] and perhaps, we are indebted [illegible] believe, that [illegible] has proceeded more from inattention [illegible] mistakes, than from any design to injure the public.

[illegible]

London [illegible] number of [illegible] can be [illegible] This is [illegible] been [illegible] by others, that [illegible] method of calculating

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The [illegible] Royal Society [illegible] has been published in the Philosophical Transactions, Vol. 59. [illegible] were republished with several additions, particularly, the [illegible]

ESSAY I.*

Containing Obſervations on the Expectations *of Lives; the Increaſe of Mankind; the Number of Inhabitants in* LONDON; *and the Influence of great Towns, on Health and Population.*

In a LETTER *to* BENJAMIN FRANKLIN, *Eſq;* L L. D. *and* F. R. S.

DEAR SIR,

I Beg leave to ſubmit to your peruſal the following obſervations. If you think them of any importance, I ſhall be obliged to you for communicating them to the Royal Society. You will find, that the chief ſubject of them is the preſent ſtate of the city of *London,* with reſpect to healthfulneſs and number of inhabitants, as far as it can be collected from the bills of mortality. This is a ſubject that has been conſidered by others; but the proper method of calculating

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* This Eſſay was read to the ROYAL SOCIETY, April 27th, 1769, and has been publiſhed in the Philoſophical Tranſactions, Vol. 59. It is here republiſhed with ſeveral additions; particularly, the *Poſtſcript.*

from

from the bills has not, I think, been ſufficiently explained.

No competent judgment can be formed of the following obſervations, without a clear notion of what the writers on *Life-Annuities* and *Reverſions* have called the *Expectation of Life*. Perhaps this is not in common properly underſtood; and Mr. *De Moivre*'s manner of expreſſing himſelf about it is very liable to be miſtaken.

The moſt obvious ſenſe of the *expectation* of a given life is, "That particular number "of years which a life of a given age has an "equal chance of enjoying." This is properly the time that a perſon may reaſonably *expect* to live; for the chances *againſt* his living longer are greater than thoſe *for* it; and, therefore, he cannot entertain an *expectation* of living longer, conſiſtently with probability. This period does not coincide with what the writers on Annuities call the *expectation of life*, except on the ſuppoſition of an uniform decreaſe in the probabilities of life, as Mr. *Simpſon* has obſerved in his *Select Exerciſes*, p. 273.—It is neceſſary to add, that, even on this ſuppoſition, it does not coincide with what is called the *expectation of life*, in any caſe of joint lives. Thus, two lives of 40 have an even chance, according to Mr. *De Moivre*'s hypotheſis (*a*), of continuing together only 13½ years. But the *expectation*

(*a*) See the Notes in pag. 2, and 23.

of two equal joint lives being (according to the ſame hypotheſis) always a *third* of the *common complement*; it is in this caſe $15\frac{1}{3}$ years. It is neceſſary, therefore, to obſerve, that there is another ſenſe of this phraſe, which ought to be carefully diſtinguiſhed from that now mentioned. It may ſignify, "The "*mean continuance* of any given *ſingle*, *joint*, "or *ſurviving* lives, according to any given "Table of obſervations:" that is, the number of years which, taking them one with another, they actually enjoy, and may be conſidered as ſure of enjoying; thoſe who live or ſurvive *beyond* that period, enjoying as much *more* time in proportion to their number, as thoſe who *fall ſhort* of it enjoy *leſs*. Thus; Suppoſing 46 perſons alive, all 40 years of age; and that, according to Mr. *De Moivre's hypotheſis*, one will die every year 'till they are all dead in 46 years; half 46, or 23, will be their *expectation of life*: That is; The number of years enjoyed by them all, will be juſt the ſame as if every one of them had lived 23 years, and then died; ſo that, ſuppoſing no intereſt of money, there would be no difference in value between annuities payable for life to every ſingle perſon in ſuch a ſet, and equal annuities payable to another equal ſet of perſons of the ſame common age, ſuppoſed to be all ſure of living juſt 23 years and no more.

In like manner; the *third* of 46 years, or 15 years and 4 months (*a*), is the *expectation* of two joint lives both 40; and this is alſo the *expectation* of the ſurvivor. That is; ſuppoſing a ſet of marriages between perſons all 40, they will, one with another, laſt juſt this time; and the ſurvivors will laſt the ſame time. And annuities payable during the continuance of ſuch marriages would, ſuppoſing no intereſt of money, be of exactly the ſame value with annuities to begin at the extinction of ſuch marriages, and to be paid, during life, to the ſurvivors. In adding together the years which any great number of ſuch marriages, and their ſurvivorſhips have laſted, the ſums would be found to be equal.

One is naturally led to underſtand the *expectation* of life in the firſt of the ſenſes now explained, when, by Mr. *Simpſon* and Mr. *De Moivre*, it is called, *the number of years which, upon an equality of chance, a perſon may expect to enjoy*; or, *the time which a perſon of a given age may juſtly expect to continue in being*; and, in the laſt ſenſe, when it is called, *the ſhare of life due to a perſon*. But, as in reality it is always uſed in the laſt of theſe ſenſes, the former language ſhould not be appplied to it: And it is in this laſt ſenſe, that it coincides with the *ſums* of the *preſent* probabilities, that any given ſingle or joint lives ſhall attain to the end of the

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(*a*) See Note (L) Appendix.

1ſt, 2d, 3d, &c. *moments*, from this time to the end of their poſſible exiſtence; or, (in the caſe of ſurviorſhips) with the ſum of the probabilities, that there ſhall be a ſurvivor at the end of the 1ſt, 2d, 3d, &c. moments, from the preſent time to the end of the poſſible exiſtence of ſurvivorſhip. This coincidence every one converſant in theſe ſubjects muſt ſee, upon reflecting, that both theſe ſenſes give the true preſent value of a life-annuity, ſecured by land, without intereſt of money (*a*).

This period in joint lives, I have obſerved is never the ſame with the period which they have an equal chance of enjoying; and in ſingle lives, I have obſerved, they are the ſame only on the ſuppoſition of an uniform decreaſe in the probabilities of life. If this decreaſe, inſtead of being always uniform, is *accelerated* in the laſt ſtages of life; the former period, in ſingle lives, will be *leſs* than the latter; if *retarded*, it will be *greater*.

It is neceſſary to add, that the number expreſſing the former period, multiplied by the number of ſingle or joint lives whoſe expectation it is, added annually to a ſociety or town, gives the whole number living together, to which ſuch an annual addition would in time grow. Thus; ſince 19, or the third of 57, is the *expectation* of two

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(*a*) See Note (L) in the Appendix.

joint lives whoſe common age is 29, or common *complement* 57; twenty marriages every year between perſons of this age would, in 57 years, grow to 20 times 19, or 380 marriages always exiſting together. The number of *ſurvivors* alſo ariſing from theſe marriages, and always living together, would, in twice 57 years, increaſe to the ſame number. And, ſince the *expectation* of a ſingle life is always half its *complement*; in 57 years likewiſe, 20 ſingle perſons aged 29, added annually to a town, would increaſe to 20 times 28.5 or 570; and, when arrived at this number, the deaths every year will juſt equal the acceſſions, and no further increaſe be poſſible.

It appears from hence, that the particular proportion that becomes extinct every year, out of the whole number conſtantly exiſting together of ſingle or joint lives, muſt, wherever this number undergoes no variation, be exactly the ſame with the *expectation* of thoſe lives, at the time when their exiſtence commenced. Thus; was it found that a 19th part of all the marriages among any body of men, whoſe numbers do not vary, are diſſolved every year by the deaths of either the huſband or wife, it would appear that 19 was, at the time they were contracted, the *expectation* of theſe marriages. In like manner; was it found in a ſociety, limited to a fixed number of members, that a 28th part dies annually out of the whole number of members, it would appear that 28 was their common expectation of life at the time they entered. So likewiſe; were it found in any town or diſtrict, where the number of births and burials are equal, that a 20th or 30th part of the inhabitants die annually, it would appear, that 20 or 30 was the *expectation* of a child juſt born in that town or diſtrict. Theſe *expectations*, therefore, for all *ſingle* lives, are eaſily found by a *Table of Obſervations*, ſhewing the number that die annually at all ages, out of a given number alive at thoſe ages; and the general rule for this purpoſe, is "to divide "the ſum of all the living in the Table, at "the age whoſe expectation is required, and "at all greater ages, by the ſum of all that "die annually at that age, and above it; or, "which is the ſame, by the number in the "Table of the living at that age; and half "unity ſubtracted from the quotient will be "the required *expectation* (*a*)." Thus, in Dr. *Halley*'s Table, the ſum of all the living at 20 and upwards is, 20,724. The number living at that age is 598; and the former number divided by the latter, and half unity

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(*a*) This rule, and alſo rules for finding in all caſes the expectations of joint lives and ſurvivorſhips, may be deduced with great eaſe, by having recourſe to the doctrine of fluxions. In this method, Mr. *De Moivre* ſays, he diſcovered them. See Appendix, Note (L), where an account will be given of theſe deductions, omitted by Mr. *De Moivre*.

(*a*) ſubtracted from the quotient, gives 34.15 for the *expectation* of 20. The expectation of the ſame life by Mr. *Simpſon*'s Table, formed from the bills of mortality of London, is 28.9 (*b*).

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(*a*) If we conceive the *recruit* neceſſary to ſupply the *waſte* of every year to be made always at the *end* of the year, the *dividend* ought to be the *medium* between the numbers living at the *beginning* and the *end* of the year. That is, it ought to be taken *leſs* than the ſum of the living in the Table at and above the given age, by *half* the number that die in the year; the effect of which *diminution* will be the ſame with the *ſubtraction* here directed.—The reaſon of this ſubtraction will be further explained, in the beginning of the laſt Eſſay.

(*b*) It appears in p. 169 and 170, that the *expectations* of *ſingle* and *joint* lives are the ſame with the values of *annuities* on theſe lives, ſuppoſing no intereſt or improvement of money.—In conſidering this ſubject, it will, probably, occur to ſome, that, allowing intereſt for money, the values of lives muſt be the ſame with the values of annuities *certain* for a number of years equal to the *expectations* of the lives. But care muſt be taken not to fall into this miſtake. The latter values are always greater than the former: And the reaſon is, that, tho' a number of *ſingle* or *joint* lives of given ages will, among them, enjoy a *given* number of years, yet ſome of them will enjoy a much *greater*, and ſome a much *leſs* number of years. Thus; 100 marriages among perſons, all 29, would, as I have ſaid, one with another, exiſt 19 years; and an office bound to pay annuities to ſuch marriages during their continuance, might reckon upon making 19 payments for each marriage. But then, many of theſe payments would not be made 'till the end of 30, and ſome not 'till the end of 40 years. And it is apparent, that on account of the greater value of *quick* than *late* payments, when money *bears* intereſt, 19 payments ſo made cannot be worth as much, as the ſame number of payments made

regularly

Theſe obſervations bring me to the principal point which I have had all along in view. They ſuggeſt to us an eaſy method of finding the number of inhabitants in a place, from a *Table of Obſervations*, or the *bills of mortality* for that place, ſuppoſing the yearly births and burials equal. "Find by "the Table, in the way juſt deſcribed, the "*expectation* of an infant juſt born, and this, "multiplied by the number of yearly births, "will be the number of inhabitants." At *Breſlaw*, according to Dr. *Halley*'s Table, though half die under 16, and therefore an infant juſt born has an *equal chance* of living only 16 years; yet his *expectation*, found by the rule I have given, is near 28 years; and this, multiplied by 1238 the number born annually, gives 34,664, the number of inhabitants. In like manner, it appears from

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regularly at the end of every year, 'till in 19 years they are all made.

This obſervation might be employed, to demonſtrate further, the error of thoſe who have maintained, that the value of a given life is the ſame, with the value of an annuity certain, for as many years as the life has an equal chance of exiſting. Were this true, an annuity on a life, ſuppoſed to be expoſed to ſuch danger in a particular year, as to create an equal chance, whether it will not fail that year, would, at the beginning of the year, be worth *nothing*, though ſuppoſed to be ſure of continuing for ever, if it eſcaped that danger.--But there can be no occaſion for taking notice of an opinion, which has been embraced only by perſons ignorant of mathematics, and plainly unacquainted with the genuine principles of calculation on this ſubject.—See a Pamphlet on Life-Annuities by *Weyman Lee*, Eſq; of the *Inner Temple.*

Mr. *Simpſon*'s Table, that, though an infant juſt born in *London* has not an *equal chance* of living 3 years, his *expectation* is 20 years; and this number, multiplied by the yearly births, would give the number of inhabitants in *London*, were the births and burials equal.--The medium of the yearly births, for 10 years, from 1759 to 1768, was 15,710. This number multiplied by 20, is 314,200; which is the number of inhabitants that there would be in *London*, according to the bills, were the yearly burials no more than equal to the births: that is, were it to ſupport itſelf in its number of inhabitants, without any ſupply from the country. But for the period I have mentioned, the burials were, at an average, 22,956, and exceeded the chriſtenings 7,246. This is, therefore, at preſent, the yearly addition of people to *London* from other parts of the kingdom, by whom it is kept up. Suppoſe them to be all, one with another, perſons who have, when they remove to *London*, an *expectation* of life equal to 30 years. That is; ſuppoſe them to be all of the age of 18 or 20, a ſuppoſition certainly far beyond the truth. From hence will ariſe, according to what has been before obſerved, an addition of 30 multiplied by 7.246; that is, 217,380 inhabitants. This number, added to the former, makes 531,580; and this, I think, at moſt, would be the number of inhabitants in *London* were the bills perfect.

But it is certain, that they give the number of births and burials too little. There are many burying-places that are never brought into the bills. Many alſo emigrate to the navy and army and country; and theſe ought to be added to the number of deaths. What the deficiencies ariſing from hence are, cannot be determined. Suppoſe them equivalent to 6000 every year in the births, and 6000 in the burials. This would make an addition of 20 times 6000, or 120,000, to the laſt number; and the whole number of inhabitants would be 651,580. If the burials are deficient only two-thirds of this number, or 4000; and the births, the whole of it; 20 multiplied by 6000, muſt be added to 314,290, on account of the defects in the births: And, ſince the exceſs of the burials above the births will then be only 5,246; 30 multiplied by 5,246 or 157,380, will be the number to be added on this account; and the ſum, or number of inhabitants, will be 591,580.—But if, on the contrary, the burials are deficient 6000, and the births only 4000; 80,000 muſt be added to 314,290, on account of the deficiencies in the births; and 30 multiplied by 9,246, or 277,380, on account of the exceſs of the burials above the births; and the whole number of inhabitants will be 671,580.

Every ſuppoſition in theſe calculations is too high. *Emigrants* from *London* are, in particular, allowed the ſame *expectation* of continuance in *London* with thoſe who are born in it, or who come to it in the firmeſt part of life, and never afterwards leave it; whereas it is not credible that the former *expectation* ſhould be ſo much as half the latter. But I have a further reaſon for thinking that this calculation gives too high numbers, which has with me irreſiſtible weight. It has been ſeen, that the number of inhabitants comes out leſs on the ſuppoſition, that the defects in the chriſtenings are greater than thoſe in the burials. Now it ſeems evident that this is really the caſe; and, as it is a fact not attended to, I will here endeavour to explain diſtinctly the reaſon which proves it.

The proportion of the number of births in *London*, to the number who live to be 10 years of age, is, by the bills, 16 to 5. Any one may find this to be true, by ſubtracting the *annual medium* of thoſe who have died under 10, for ſome years paſt, from the *annual medium* of births for the ſame number of years.——Now, tho', without doubt, *London* is very fatal to children, yet it ſeems incredible that it ſhould be ſo fatal as this implies. The *bills*, therefore, probably, give the number of thoſe who die under 10 too great in proportion to the number of births; and there can be no other cauſe of this, than a greater deficiency in the *births* than in the *burials.*

burials. Were the deficiencies in both equal; that is, were the *burials*, in proportion to their number, juſt as deficient as the *births* are in proportion to *their* number, the proportion of thoſe who reach 10 years of age to the number born, would be right in the *bills*, let the deficiencies themſelves be ever ſo conſiderable. On the contrary; were the deficiencies in the *burials* greater than in the *births*, this proportion would be given too great; and it is only when the former are leaſt, that this proportion can be given too little.——Thus; let the number of annual *burials* be 23,000; of *births* 15,700; and the number dying annually under 10, 10,800. Then 4,900 will reach 10, of 15,700 born annually; that is, 5 out of 16. —Were there no deficiencies in the *burials*, and were it fact that only *half* the number born die under 10; it would follow, that there was an annual deficiency equal to 4,900 ſubtracted from 10,800, or 5,900, in the *births*.—Were the *births* a third part too little, and the *burials* alſo a third part too little, the true number of *births*, *burials*, and of *children dying under* 10, would be 20,933--30,666, and 14,400; and, therefore, the number that would live to 10 years of age, would be 6,533 out of 20,933, or 5 of 16 as before.—Were the *births* a third part, and the *burials* ſo much as two-fifths wrong, the number of *births*, *burials*, and children dying under 10 would be 20,933—32,200—and 15,120. And, therefore, the number that would live to 10 would be 5,813 out of 20,933, or five out of 18.—Were the *births* a 3d part wrong, and the *burials* but a 6th, the foregoing numbers would be 20,933—26,833—12,600; and, therefore, the number that would live to 10 would be 8,333 out of 20,933, or 5 out of 12.56: And this proportion ſeems as low as is conſiſtent with probability. It is ſomewhat leſs than the proportion in Mr. *Simpſon's* Table of *London Obſervations*; and much leſs than the proportion in the Table of *Obſervations* for *Breſlaw*. The deficiencies, therefore, in the *births* muſt be greater than thoſe in the *burials* (*a*); and the leaſt number I have given, or 591,580 is neareſt to the true number of inhabitants. However, ſhould any one, after all, think that it is not improbable that only 5 of 16 ſhould live in *London* to be 10 years of age; or that above *two-thirds* die under this age; the conſequence will ſtill be, that the foregoing calculation

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(*a*) One obvious reaſon of this fact is, that *none* of the *births* among *Jews*, *Quakers*, *Papiſts*, and the *three denominations of Diſſenters* are included in the bills, whereas *many* of their *burials* are. It is further to be attended to, that the abortive and ſtill-born, amounting to about 600 annually, are included in the burials, but never in the births. If we add theſe to the chriſtenings, preſerving the burials the ſame, the proportion of the born, according to the bills, who have reached ten for the laſt ſixteen years, will be very nearly one *third* inſtead of *five ſixteenths*.

culation has been carried too high. For it will from hence follow, that the *expectation* of a child juſt born in *London* cannot be ſo much as I have taken it. This *expectation* is 20, on the ſuppoſition that half die under 3 years of age, and that 5 of 16 live to be 29 years of age, agreeably to Mr. *Simpſon*'s Table. But if it is indeed true, that *half* die under 2 years of age, and 5 of 16 under 10, agreeably to the *bills*, this expectation cannot be ſo much as 17 (*a*); and all the numbers before given will be conſiderably reduced.

Upon the whole: I am forced to conclude from theſe obſervations, that the ſecond number I have given, or 651,580, though ſhort of the number of inhabitants commonly ſuppoſed in *London*, is, very probably, much *greater*, but cannot be *leſs*, than the true number. Indeed, it is in general evident, that in caſes of this kind numbers are very much over-rated. The ingenious Dr. *Brakenridge*, 14 years ago, when the bills were lower than they are now, from the number of houſes, and allowing ſix to a houſe, made the number of inhabitants 751,800. But his method of determining the (*b*) number of houſes is too precarious;

(*a*) This may be deduced from the obſervations in the laſt Eſſay; and it will be there proved, that, in reality, this expectation does not exceed 18.

(*b*) Vid. Phil. Tranſactions, Vol. XLVIII, p. 788. In a paper ſubſequent to this, Dr. *Brakenridge* tells us, that carious; and, beſides, ſix to a houſe is too large an allowance.—Many families now have two houſes to live in.—The magiſtrates of *Norwich*, in 1752, took an exact account of both the number of houſes and individuals in that city. (*a*) The number of houſes

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that in a late ſurvey it appeared, that in all *Middleſex*, *London*, *Weſtminſter*, and *Southwark*, there were 87,614 houſes, of which 19,324 were cottages, and 4810 empty. And he acknowledges, that this, if right, proves *London* to be much leſs populous than he had made it. See Phil. Tranſ. Vol. 50, p. 471. He does not mention how this ſurvey was taken; but moſt probably it muſt have been incorrect.—Mr. *Maitland* gives two accounts of the number of houſes within the bills. One carefully taken from the books of all the pariſhes and precincts belonging to *London*; and another taken from a particular ſurvey in 1737, made by himſelf with incredible pains. The firſt account makes the number of houſes 85,805. The ſecond account makes it 95,968. And the reaſon of the difference he obſerves, is, that many landlords of ſmall places, paying all taxes, they are in the pariſh books reckoned as ſo many ſingle houſes, tho' each of them contain ſeveral houſes. See Mr. *Maitland*'s Hiſtory of *London*, 2d Book at the end.—This, perhaps, may be alſo the reaſon of the deficiencies which, I ſuppoſe, there muſt be in the ſurvey, mentioned by Dr. *Brakenridge*.—It will be obſerved preſently, that the number of inhabitants in *London* in 1737, was conſiderably greater than it is now.

(*a*) Vid. Gentleman's Magazine for 1752, and Dr. *Short's Comparative Hiſtory of the Increaſe of Mankind*, p. 38. In page 58 of this laſt work the author ſays, that, in order to be fully ſatisfied about the number of perſons to be allowed to a family, he procured the true number of families and individuals in 14 market towns, ſome of them conſiderable for trade and populouſneſs; and that in them were 20,371 families, and 97,611 individuals, or houſes was 7,139, and of individuals 36,169, which gives nearly 5 to a houſe.——Ano-

ther

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or but little more than $4\frac{3}{4}$ to a family. He adds, that, in order to find the difference in this reſpect between towns of trade and country pariſhes, he procured, from divers parts of the kingdom, the exact number of *families* and *individuals*, in 65 country pariſhes. The number of *families* was 17,208; *individuals* 76,284; or not quite $4\frac{1}{2}$ to a family.—In the place I have juſt referred to, in the Gentleman's Magazine, there is an account of the number of *houſes* and *inhabitants* in *Oxford*, excluſive of the colleges; and in *Wolverhampton*, *Coventry* and *Birmingham*, for 1750. The number of perſons to a *houſe* was, by this account, $4\frac{4}{5}$ in the two former towns, and $5\frac{3}{4}$ in the two latter.—Dr. *Davenant*, from Mr. *King*'s Obſervations, gives $4\frac{1}{13}$, as the number of perſons to a *family* for the whole kingdom. See *An Eſſay on the probable method of making a people gainers by the balance of trade.* From an account with which a friend at *Shrewſbury* has favoured me, it appears, that in that town, in 1750, the number of inhabitants to a *houſe* was $4\frac{1}{3}$.—Very exact accounts, of which I ſhall take further notice, prove, that in the pariſh of *Holy-Croſs*, one of the ſuburbs of *Shrewſbury*, and at *Northampton*, the ſame proportion is $4\frac{1}{3}$ to a *houſe* in the former; and $4\frac{3}{4}$ in the latter.—It ſeems, therefore, that five perſons to a houſe is an allowance large enough for *London*, and too large for *England* in general. From whence it will follow, that Dr. *Brakenridge* has likewiſe over-rated the number of people in *England.* In a letter to *George Lewis Scott*, Eſq; publiſhed in 1756, in the Phil. Tranſ. Vol. 49, p. 877; he ſays, that he had been certainly informed, that the number of houſes rated to the window-tax was 690,000. The number of cottages not rated, he adds, was not accurately known; but from the accounts given in, it appeared, that they could not exceed 200,000; and from theſe data, in conſequence of allowing ſix to a houſe, he makes the number of people in *England* to be 5,340,000. Perhaps the number of houſes in this account is too little.

Suppoſe

ther method which Dr. *Brakenridge* took to determine the number of inhabitants in *London* was from the annual number of burials, adding 2000 to the bills for omiſſions, and ſuppoſing a 30th part to die every year. In order to prove this to be a moderate ſuppoſition he obſerves that, according to Dr. *Halley*'s Obſervations, a 34th part die every year at *Breſlaw*. But this obſervation was made too inadvertently. The number of annual burials there, according to Dr. *Halley*'s account, was 1174, and the number of inhabitants, as deduced by him from his Table, was 34,000; and therefore a 29th part died every year. Beſides; any one may find, that in reality the Table is conſtructed on the ſuppoſition, that the whole number born, or 1238, die every year; from whence it will follow that a 28th part died every year. (*a*) Dr. *Brakenridge*, therefore, had

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Suppoſe it a million; and let five be allowed to a houſe; and the number of people in *England* will be five millions: Which, ſince five to a houſe is too large an allowance, ought to be conſidered as, probably, *more* than the true number. — The number of people in *Scotland* and *Ireland*, Dr. *Brakenridge* eſtimates at three millions. See Phil. Tranſ. Vol. 50, p. 473.

(*a*) Care ſhould be taken, in conſidering Dr. *Halley*'s Table, not to take the firſt number in it, or 1000, for ſo many juſt born. 1238, he tells us, was the annual medium of births, and 1000 is the number he ſuppoſes all living at one year and under. It was inattention to this that led Dr. *Brakenridge* to his miſtake.

had he attended to this, would have ſtated a 24th part as the proportion that dies in *London* every year, and this would have taken off 150,000 from the number he has given. But even this muſt be leſs than the juſt proportion. For let three-fourths of all who either die in *London* or migrate from it, be ſuch as have been born in *London*; and let the reſt be perſons who have removed to *London* from the country, or from foreign nations. The *expectation* of the former, it has been ſhewn, cannot exceed 20 years; and 30 years have been allowed to the latter. One with another, then, they will have an *expectation* of 22½ years. That is; one of 22½ will die every year. (*a*) And, conſequently,

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It will be ſhewn in the 4th Eſſay, that the number of the living, under 20, is given too high in this Table; and from hence it will follow, that more than a 28th part of the inhabitants die at *Breſlaw* annually.

(*a*) The whole number of inhabitants in *Rome* in 1743, was 147,476, and the annual medium of burials for three years, from 1741 to 1743, was 6338. A 23d part, therefore, died every year. See a Treatiſe in *German*, on the different degrees of human mortality in different ſituations, by *Suſmilch*, firſt counſellor of his *Pruſſian* Majeſty's Conſiſtory, and member of the Royal Academy of ſciences at *Berlin*, p. 15.

In 1761, the whole number of inhabitants in the ſame town, was 157,452. The annual medium of births, for three years, from 1759 to 1761, was 5167; and of burials 7153. One in 22, therefore, died annually. See Dr. *Short's Comparative Hiſtory of the Increaſe and Decreaſe of Mankind in* England *and ſeveral Countries abroad*, p. 59, 60.

quently, ſuppoſing the annual recruit from the country to be 7000, the number of

births

60.—In 1752, the accurate and diligent Mr. *Struyk*, took particular pains to determine the number of inhabitants in *Amſterdam*; and the reſult of his enquiry was, that very probably it did not amount to 200,000. The annual medium of burials for ſix years, from 1747 to 1752, was 8247. *One* in 24, therefore, died annually. See *Suſmilch* ibid.—At *Amſterdam*, there is a great number of Jews, and their burials are not included in the bills. There muſt, I ſuppoſe, be other deficiencies, and an allowance for theſe would, I doubt not, increaſe the proportion of inhabitants, who die annually, to one in 21 or 22.—At *Dublin*, in the year 1695, the number of inhabitants was found, by an exact ſurvey, to be 40,508, (ſee Philoſ. Tranſactions, N° 261). I find no account of the annual burials juſt at that time; but from 1661 to 1681, the medium had been 1613; and from 1715 to 1728 it was 2123. There can, therefore, be no material error in ſuppoſing that, in 1695, it was 1800; and this makes 1 in 22 to die annually.—In 1745 the number of *families* in the ſame city appeared, by an exact account laid before the Lord Mayor, to be 9,214. It is highly probable this number of families did not conſiſt of ſo many as 45,000 individuals. Suppoſe them, however, 50,000; and, as at this time the medium of annual burials appears to have been 2,360, 1 in 21 died annually: ſee Dr. *Short*'s *Comparative Hoſtory*, p. 15, and *New Obſervations*, p. 228.—Theſe facts prove that I have been very moderate in making only 1 in $22\frac{1}{2}$, including emigrants, to die in *London* annually.—In 1631 the number of people in the *city and liberties* of *London* was taken, by order of the Privy Council, and found to be 130,178.—This account was taken five years after a plague that had ſwept off near a quarter of the inhabitants; and when, therefore, the town being full of recruits in the vigour of life, the medium of annual burials muſt have been lower than uſual, and the births higher. Could, therefore, the medium

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births 3 times 7000 or 21,000, and the *burials* and *migrations* 28,000 (which are all high ſuppoſitions), the number of inhabitants will be, $22\frac{1}{2}$ multiplied by 28,000, or 630,000.

I will juſt mention here one other inſtance of exaggeration on the preſent ſubject.

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dium of annual burials at that time, within the walls, and in the 16 pariſhes without the walls, be ſettled, excluſive of thoſe who died in ſuch parts of the 16 pariſhes without the walls, as are not in the *liberties*, the proportion dying annually obtained from hence might be depended on, as leſs than the common and juſt proportion. But this medium cannot be diſcovered with any accuracy. *Graunt* eſtimates that two-thirds of theſe 16 pariſhes are within the *liberties*; and, if this is right, the medium of annual burials in the *city and liberties* in 1631, was 5,500, and 1 in $23\frac{3}{4}$ died annually; or, making a ſmall allowance for deficiencies in the bills, 1 in 22. — Mr. *Maitland*, in his Hiſtory of *London*, Vol. II. p. 744, by a laborious, but too unſatisfactory, inveſtigation, reduces this proportion to 1 in $24\frac{1}{2}$; and on the ſuppoſitions, that this is the true proportion dying annually, *at all times*, in *London*, and that the deficiencies in the burials (including the burials in *Marybone* and *Pancraſs* pariſhes) amount to 3,038 annually; he determines, that the number of inhabitants within the bills was 725,903, in the year 1737.

The number of burials not brought to account in the bills is, probably, now much greater than either Dr. *Brakenridge* or Mr. *Maitland* ſuppoſe it. I have reckoned it ſo high as 6000, in order to include emigrants, and alſo to be more ſure of not falling below the truth.

It will appear in the laſt Eſſay, with an evidence little ſhort of demonſtration, that, at leaſt, 1 in $20\frac{3}{4}$ die annually in *London*, and that, conſequently, the number of inhabitants, if the omiſſions in the burials are 6000, cannot exceed 601,750.

Mr. *Corbyn Morris*, in his uſeful *Obſervations on the paſt growth and preſent ſtate of the city of London*, publiſhed in 1751, ſuppoſes that no more than a 60th part of the inhabitants of *London*, who are above 20, die every year, and from hence he concludes that the number of inhabitants was near a million. In this ſuppoſition there was an error of at leaſt one half. According to Dr. *Halley*'s Table, it has been ſhewn, that a 34th part of all at 20 and upwards, die every year at *Breſlaw*. In *London*, a 29th part, according to Mr. *Simpſon*'s Table, and alſo according to all other Tables of *London* Obſervations. And in *Scotland* it has been found for many years, that, of 974 miniſters and profeſſors whoſe ages are 27 and upwards, a 33d part have died every year. Had, therefore, Mr. *Morris* ſtated a 30th part of all above 20 as dying annually in *London*, he would have gone beyond the truth, and his concluſion would have been 400,000 leſs than it is.

Dr. *Brakenridge* obſerved, that the number of inhabitants, at the time he calculated, was 127,000 leſs than it had been. The bills have lately advanced a little, but ſtill they are much below what they were from 1717 to 1743. The medium of the annual *births*, for 20 years, from 1716 to 1736, was 18,000, and of *burials* 26,529; and, by calculating from hence on all the ſame ſuppoſitions with thoſe which made 651,580 to be the preſent number of inhabitants in *London*, it will be found that the number then was 735,840, or 84,260 greater than the number at preſent. *London*, therefore, for the laſt 30 years, has been decreaſing; and though now it is increaſing again, yet there is reaſon to think that the additions lately made to the number of buildings round it, are owing, chiefly to the increaſe of luxury, and the inhabitants requiring more room to live upon (*a*).

It ſhould be remembered, that the number of inhabitants in *London* is now ſo much leſs as I have made it, than it was 40 years ago,

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(*a*) The medium of annual burials in the 97 pariſhes within the walls was,

From 1655 to 1664,	——	3264
From 1680 to 1690,	——	3139
From 1730 to 1740,	——	2316
From 1758 to 1768,	——	1620

This account proves, that though, ſince 1655, *London* has doubled its inhabitants, yet, *within the walls*, they have decreaſed; and ſo rapidly for the laſt 30 years as to be now reduced to one half.—The like may be obſerved of the 17 pariſhes immediately without the walls. Since 1730, theſe pariſhes have been decreaſing ſo faſt, that the *annual burials* in them have ſunk from 8,672 to 5,432, and are now lower than they were before the year 1660. In *Weſtminſter*, on the contrary, and the 23 out-pariſhes in *Middleſex* and *Surrey*, the *annual burials* have ſince 1660, advanced from about 4000 to 16,000. —— Theſe facts prove, that the inhabitants of *London* are now much leſs crowded together than they were. It appears, in particular, that *within the walls* the inhabitants take as much room to live upon as double their number did formerly. —The very ſame concluſions may be drawn from an examination of the *chriſtenings*.

ago, on the ſuppoſition, that the proportion of the omiſſions in the *births* to thoſe in the *burials*, was the ſame then that it is now. But it appears that this is not the fact.—From 1728, (the year when the ages of the dead were firſt given in the *bills*) to 1742, near five-ſixths of thoſe who were born died under 10, according to the bills. From 1742 to 1752 three quarters: And ever ſince 1752, this proportion has ſtood nearly as it is now, or at ſomewhat more than two-thirds. The omiſſions in the *births*, therefore, compared with thoſe in the *burials*, were greater formerly; and this muſt render the difference between the number of inhabitants now and formerly leſs conſiderable than it may ſeem to be from the face of the bills. One reaſon, why the proportion of the amounts of the *births* and *burials* in the bills, comes now nearer than it did, to the true proportion, may, perhaps, be, that the number of Diſſenters is leſſened. The Foundling Hoſpital alſo may have contributed a little to this event, by leſſening the number given in the bills as having died under 10, without taking off any from the *births*; for all that die in this hoſpital are buried at *Pancraſs* church, which is not within the *bills*. See the preface to a collection of the yearly bills of mortality from 1657 to 1758 incluſive, p. 15.

I will add, that it is probable that *London* is now become leſs fatal to children than it was; and that this is a further circumſtance which muſt reduce the difference I have mentioned; and which is likewiſe neceſſary to be joined to the greater deficiencies in the births, in order to account for the very ſmall proportion of children who ſurvived 10 years of age, during the two firſt of the periods I have ſpecified.—Since 1752, *London* has been thrown more open. The cuſtom of keeping country-houſes, and of ſending children to be nurſed in the country, has prevailed more. But, particularly, the deſtructive uſe ſpirituous liquors among the poor has been checked.

I have ſhewn that in *London*, even in its preſent ſtate, and according to the moſt moderate computation, half the number born die under *three* years of age. But it appears from *Graunt*'s (*a*) accurate account of the births, weddings, and burials in three country pariſhes for 90 years; and alſo, from Dr. *Short*'s collection of obſervations in his *Comparative Hiſtory*, and his treatiſe entitled, *New Obſervations on Town and Country Bills*

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(*a*) See *Natural and Political Obſervations on the Bills of Mortality*, by Capt. *John Graunt*, F. R. S.—See alſo Mr. *Derham's Phiſico-Theology*, p. 174, where it appears, that in the pariſh of *Aynho* in *Northamptonſhire*, tho' the *births* had been, for 118 years, to the marriages as 6 to 1; yet the *burials* had been to the marriages only as 3¾ to 1.

of Mortality; that in country villages and pariſhes, the major part live to mature age; and even to marry.

In the pariſh of *Holy-Croſs* (*a*), in *Salop*, it appears from a curious regiſter, which has been kept by the Rev. Mr. *Gorſuch*, the vicar, that of 655 who have died there at all ages for the laſt 20 years, 321, or near one half, have lived to 30 years of age: And, by forming a Table of Obſervations from this regiſter,

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(*a*) This pariſh contains in it a village which is a part of the ſuburbs of *Shrewſbury*. It conſiſts of 1400 acres of arable and paſture land; beſides 300 acres taken up by houſes and gardens. It is ſix miles in circumference; half of which lies along the banks of the river *Severn*.—I mention theſe particulars to ſhew, that it may be reckoned a *country* pariſh; tho', perhaps, not perfectly ſo, on account of its nearneſs to *Shrewſbury*.—The chriſtenings in it exceed the burials a little; and the number of inhabitants (moſtly labouring people) has, for the laſt 20 years, kept nearly to 1050, without any conſiderable increaſe.—The regiſter of this pariſh, from 1750 to 1760, has been publiſhed in the 52d volume of the *Philoſophical Tranſactions*, Part I. Art. 25. And a continuation of it from 1760 to 1770, has been lately communicated and read to the Royal Society. It is to be wiſhed, that more ſuch accounts, ſpecifying, as this does, the males and females dying at all ages, were kept in different ſituations in the country. This is the only one that I have ever heard of. It is kept with particular care and accuracy by Mr. *Gorſuch*; and furniſhes very uſeful *data* for determining the difference in value between town and country lives.—It deſerves to be mentioned particularly, that no *foreigners* or *ſtrangers*, who happen to die in this pariſh, or who may be brought into it to be buried, are entered into the regiſter: Nor are any of the fixed inhabitants omitted, tho' carried out to be buried.

regiſter, in the manner which will be deſcribed in the laſt Eſſay, I find that a child juſt born in this pariſh has an expectation of 33 years; and that in general, under the age of 50, the *expectations* of lives here exceed thoſe in *London*, in the proportion of about 4 to 3.—So great is the difference, eſpecially to children, between living in great towns and in the country.—But nothing can place this obſervation in a more ſtriking light, than the account given by Dr. *Thomas Heberden*, and publiſhed in the Philoſophical Tranſactions (vol. LVII. p. 461), *of the increaſe and mortality of the inhabitants of the iſland of Madeira.* In this iſland, it ſeems, the weddings have been to the births, for 8 years, from 1759 to 1766, as 10 to 46.8; and to the burials, as 10 to 27.5. Double theſe proportions, therefore, or the proportion of 20 to 46.8, and of 20 to 27.5, are the proportions of the number marrying annually, to the number born and the number dying. Let 1 marriage in 10 be a 2d or 3d marriage on the ſide of either the man or the woman; and 10 marriages will imply 19 individuals who have grown up to maturity, and lived to marry once or oftener; and the proportion of the number marrying annually the firſt time, to the number dying annually, will be 19 to 27.5, or near 3 to 4. It may ſeem to follow from hence, that in this iſland near three-fourths of thoſe who die have been married; and, conſequently, that not many more than a *quarter* of the inhabitants die in childhood and celibacy; and this would be a juſt concluſion were there no increaſe, or had the births and burials been equal. But it muſt be remembered, that the general effect of an increaſe while it is going on in a country, is to render the proportion of perſons marrying annually, to the annual deaths, *greater*, and to the annual births *leſs*, than the true proportion marrying, out of any given number born. This proportion generally lies between the other two proportions, but always neareſt to the firſt (*a*); and, in the preſent caſe,

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(*a*) In a country where there is no increaſe or decreaſe of the inhabitants, and where alſo life, in its firſt periods, is ſo ſtable, and marriage ſo much encouraged, as that half of all who are born live to be married, the *annual* births and burials muſt be equal, and alſo *quadruple* the number of weddings, after allowing for 2d and 3d marriages. Suppoſe in theſe circumſtances (every thing elſe remaining the ſame) the *probabilities of life*, during its firſt ſtages, to be improved. In this caſe, more than *half* the born will live to be married, and an increaſe will take place. The births will exceed the burials, and both fall below *quadruple* the weddings; or, which is the ſame, below *double* the number annually married.—Suppoſe next (the *probabilities of life* and the *encouragement to marriage* remaining the ſame) the *prolifickneſs* only of the marriages to be improved. In this caſe it is plain, that an increaſe alſo will take place; but the *annual* births and burials, inſtead of being leſs, will now both riſe above *quadruple* the weddings; and therefore the proportion of the born to that part of the born who marry (being by ſuppoſition two to one) will be leſs than the proportion of caſe, it is ſufficiently evident that it cannot be much leſs than two-thirds.

In

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of either the *annual* births or the *annual* burials, to the number marrying *annually*.—Suppoſe again (the *encouragement to marriage* remaining the ſame) that the *probabilities of life* and the *prolifickneſs of marriages* are both improved. In this caſe, a more rapid increaſe will take place, or a greater exceſs of the births above the burials; but at the ſame time they will keep nearer to *quadruple* the weddings, than if the latter cauſe only had operated, and produced the ſame increaſe.—I ſhould be too minute and tedious, were I to explain theſe obſervations at large. It follows from them, that, in every country or ſituation where, for a courſe of years, the *burials* have been either *equal to* or *leſs* than the *births*, and both under *quadruple* the marriages; and alſo that, wherever the burials are *leſs* than quadruple the annual marriages, and at the ſame time the births *greater*, there the major part of all that are born live to marry. In the inſtance which I have conſidered above, and which occaſions this note, the annual births are ſo much *greater* than *quadruple* the marriages, and at the ſame time the annual burials ſo much *leſs*, that the proportion that live to marry of thoſe who are born, can ſcarcely be much leſs than I have ſaid; or two-thirds.

I have ſhewn how the allowance is to be made for 2d and 3d marriages; but it is not ſo conſiderable as to be of any particular conſequence; and, beſides, it is, in part, compenſated by the natural children which are included in the births, and which raiſe the proportion of the births to the weddings higher than it ought to be, and therefore bring it nearer to the true proportion of the number born *annually*, to thoſe who marry annually, after deducting thoſe who marry a 2d or 3d time.

In drawing concluſions from the proportion of *annual* births and burials, in different ſituations, ſome writers on the increaſe of mankind, have not given due attention to the difference in theſe proportions, ariſing from the different

In *London,* then, *half* die under three years of age; and in *Madeira* about *two-thirds* of all who are born live to be married. Agreeable to this, it appears alſo from the account I have referred to, that the *expectation* of a child juſt born in *Madeira* is about 39 years; or double the expectation of a child juſt born in *London.* For the number of inhabitants was found, by a ſurvey made in the beginning of the year 1767, to be 64,614. The annual medium of *burials* had been, for eight years, 1293; of *births* 2201. The number of inhabitants, divided by the annual medium of *burials,* gives 49.89; or the *expectation* nearly of a child juſt born, ſuppoſing the *births* had been 1293, and conſtantly equal to the *burials,* the number of inhabitants remaining the ſame. And the ſame number, divided by the annual medium of *births,* gives 29.35; or the *expectation* of a child juſt born, ſuppoſing the burials 2201, the number of births

different circumſtances of increaſe or decreaſe among a people. One inſtance of this I have now mentioned; and one further inſtance of it is neceſſary to be mentioned. The proportion of *annual* births to weddings has been conſidered as giving the true number of children derived from each marriage, taking all marriages one with another. But this is true only when, for many years, the births and burials have kept nearly equal. Where there is an exceſs of the births occaſioning an increaſe, the proportion of *annual* births to weddings muſt be leſs than the proportion of children derived from each marriage; and the contrary muſt take place where there is a decreaſe.

and of inhabitants remaining the ſame. And the true *expectation* of life muſt be ſomewhere near the mean between 49.89 and 29.35.

Again: A 50th part of the inhabitants of *Madeira*, it appears, die annually. In *London*, I have ſhewn, that above twice this proportion dies annually. In ſmaller towns a ſmaller proportion dies; and the births alſo come nearer to the burials. In general; there ſeems reaſon to thing that in towns (allowing for particular advantages of ſituation, trade, police, cleanlineſs, and openneſs, which ſome towns may have,) the exceſs of the burials above the births, and the annual deaths are more or leſs as the towns are greater or ſmaller. In *London* itſelf, about 160 years ago, when it was ſcarcely a fourth of its preſent bulk, the births were much nearer to the burials, than they are now. But in country pariſhes and villages the births almoſt always exceed the burials; and I believe it ſeldom happens that ſo many as a 30th, or more than a 40th part of the inhabitants, die annually (*a*). In the four provinces

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(*a*) In the year 1733, a ſurvey was taken of the inhabitants of the pariſh of *Stoke-Damerel* in *Devonſhire*, and the number of men, women, and children was found to be 3361. — The *chriſtenings* for the year were 122— the *weddings* 28—*burials* 62.—No more, therefore, than the 54th part of the inhabitants died in the year.—In part of this year an epidemical fever prevailed in the pariſh. See Martyn's *Abridgment of the Philoſ. Tranſactions*, vol. IX, provinces of *New-England* there is a very rapid increaſe of the inhabitants; but, notwithſtanding this, at *Boſton*, the capital, the inhabitants would decreaſe were there no ſupply from the country: for, if the account I have ſeen is juſt, from 1731 to 1762, the burials all along exceeded the births (*a*). So remarkably do towns, in conſequence of their unfavourableneſs to health, and the

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IX. p. 325.—According to *Graunt*'s account of a pariſh in *Hampſhire*, not reckoned, he ſays, remarkably healthful, a 50th part of the inhabitants had died annually for 90 years. *Natural and political Obſervations, &c.* Chap. xii.

In 1098 country pariſhes, mentioned by *Suſmilch*, the annual average of deaths, for ſix years, ending in 1749, was 5255. The number of inhabitants was 225,357. One, therefore, in 43 died annually.—In 106 other pariſhes, mentioned by him, this proportion was 1 in 50. —In the Dukedom of *Wurtemberg*, the inhabitants, he ſays, are numbered every year; and from the average of five years, ending in 1754, it appeared that, taking the towns and country together, 1 in 32 died annually.—In another province, which he mentions, conſiſting of 635,998 inhabitants, 1 in 33 died annually. From theſe facts he concludes, that, taking a whole country in *groſs*, including all cities and villages, mankind enjoy among them about 32 or 33 years each of exiſtence. And this, very probably, may not be far from the truth in the preſent ſtate of moſt of the kingdoms of *Europe*. And it will follow, that a child born in a country pariſh or village, has, at leaſt, an expectation of 36 or 37 years; ſuppoſing the proportion of *country* to *town* inhabitants to be as $3\frac{1}{2}$ to 1; which, I think, this ingenious writer's obſervations prove to be nearly the caſe in *Pomerania*, *Brandenburgh*, and ſome other kingdoms.

(*a*) See a particular account of the births and burials in this town from 1731 to 1752 in the *Gentleman's Magazine* for 1753, p. 413.

luxury which generally prevails in them, check the increaſe of countries.

Healthfulneſs and Prolifickneſs are, probably, cauſes of increaſe ſeldom ſeparated. In conformity to this obſervation, it appears from comparing the births and weddings, in countries and towns where regiſters of them have been kept, that in the former, marriages, one with another, ſeldom produce leſs than four children each; generally between four and five, and ſometimes above five. But in towns ſeldom above four; generally between three and four; and ſometimes under three (*a*).

I have ſometimes heard the great number of old people in *London* mentioned, to prove its favourableneſs to health and long life. But no obſervation can be more erroneous. There ought, in reality, to be more old people in *London*, in proportion to the number of inhabitants, than in any ſmaller towns; becauſe at leaſt one quarter of its inhabitants are perſons who come in-

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(*a*) Any one may ſee what evidence there is for this, by conſulting Dr. *Short*'s two books already quoted, and the *Abridgment of the Philoſophical Tranſactions*, vol. VII. part iv. p. 46, and *Graunt*'s account already quoted, of the births, weddings and burials in three country pariſhes for 90 years; compared with ſimilar accounts in towns. In conſidering theſe accounts, it ſhould not be forgotten that allowances muſt be made for the different circumſtances of increaſe or decreaſe in a place, agreeably to the obſervation at the end of the note in page 194.

to it, from the country, in the moſt robuſt part of life, and with a much greater probability of attaining to old age, than if they had come into it in the weakneſs of infancy. But, notwithſtanding this advantage, there are much fewer perſons who attain to great ages in *London*, than in moſt other places where obſervations have been made.—At *Vienna*; of 22,704 who died in the four years 1717, 1718, 1724, 1725 (*a*), 109 reached 90 years; that is, 48 in 10,000. But in *London*, for 30 years before 1769 only 35 of the ſame number have reached this age.—At *Breſlaw* it appears, by Dr. *Halley*'s Table, that 41 of 1238 born, or a 30th part live to be 80 years of age.—In the pariſh of *All-ſaints*, in *Northampton*, an account has been kept ever ſince 1733 of the ages at which the inhabitants die; and I find that a 22d part die there turned of 80. At *Norwich* a like account has been kept; and it appears, that for the laſt 30 years, a 27th part of the inhabitants have died, turned of the ſame age.—According to Mr. *Kerſſeboom*'s Table of Obſervations, publiſhed at the end of the laſt edition of Mr. *De Moivre*'s Treatiſe on the Doctrine of Chances, a 14*th* part of all that are born live to 80.—And in the pariſh of *Holy-Croſs*, already mentioned p. 183 and p. 192,

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(*a*) Vid. Abridgment of the Philoſophical Tranſactions, vol. VII. part iv. p. 46.—It appears alſo that more than three-fifths of all who died in theſe years at *Vienna* were boys and girls, by whom, I ſuppoſe, are meant perſons under 16.

the *eleventh* part of the inhabitants live to 80 (*a*).—But in *London*, for 30 years, ending at the year 1768, only 25 of every 1000, who have died, or a 40th part, have lived to this age; which may be eaſily diſcovered, by dividing the ſum of all who have died during theſe years at all ages, by the ſum of all who have died above 80.

Among the peculiar evils to which great towns are ſubject, I might further mention the PLAGUE. Before the year 1666, this dreadful calamity laid *London* almoſt waſte once in every 15 or 20 years; and there is no reaſon to think, that it was not generally bred within itſelf. A moſt happy alteration has taken place; which, perhaps, in part is owing to the greater advantages of cleanlineſs and openneſs, which *London* has enjoyed ſince it was rebuilt; and which lately have been very wiſely improved.

The facts I have now taken notice of are ſo important that, I think, they deſerve more attention than has been hitherto beſtowed upon

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(*a*) This, however, will appear itſelf inconſiderable, when compared with the following account: "In 1761 "the burials in the diſtrict of *Chriſtianna*, in *Norway*, "amounted to 6,929, and the chriſtenings to 11,024. "Among thoſe who died, 394, or 1 in 18, had lived to "the age of 90; 63 to the age of 100, and ſeven to the "age of 101.—In the dioceſe of *Bergen*, the perſons "who died amounted only to 2,580, of whom 18 lived "to the age of 100; one woman to the age of 104, and "another woman to the age of 108."

See the *Annual Regiſter* for 1761, p. 191.

upon them. Every one knows that the ſtrength of a ſtate conſiſts in the number of people. The encouragement of population, therefore, ought to be one of the firſt objects of policy in every ſtate; and ſome of the worſt enemies of population are the luxury, the licentiouſneſs, and debility produced and propagated by great towns.

I have obſerved that *London* is now (*a*) increaſing. But it appears that, in truth, this is an event more to be dreaded than deſired. The more *London* increaſes, the more the reſt of the kingdom muſt be deſerted; the fewer hands muſt be left for agriculture; and, conſequently, the leſs muſt be the plenty and the higher the price of all the means of ſubſiſtence.—*Moderate* towns, being ſeats of refinement, emulation, and arts, may be public advantages. But *great* towns, long before they grow to half the bulk of *London*, become checks on population of too hurtful a nature, nurſeries of debauchery and voluptuouſneſs; and, in many reſpects, greater evils than can be compenſated by any advantages (*b*).

Dr.

(*a*) This increaſe is greater than the bills ſhew, on account of the omiſſion in them of the two pariſhes which have been moſt increaſed by new buildings; I mean *Mary-bone* and *Pancraſs* pariſhes. The former of theſe pariſhes is, I ſuppoſe, now one of the largeſt in *London*.

(*b*) The mean annual *births*, *weddings*, and *burials* in the following towns, for ſome of the laſt years, have been nearly,

At

Dr. *Heberden* obſerves that, in *Madeira*, the inhabitants double their own number in 84 years. But this (as you, Sir, well know) is a very ſlow increaſe, compared with that which takes place among our colonies in AMERICA. In the back ſettlements, where the inhabitants apply themſelves entirely to agriculture, and luxury is not known, they double their own number in 15 years; and all thro' the northern colonies, in 25 years (*a*). This is an inſtance of increaſe ſo rapid, as to have ſcarcely any parallel. The births in theſe countries muſt exceed the burials much more than in *Madeira*; and a greater proportion of the born muſt reach maturity.—In 1738, the number of inhabitants in *New Jerſey* was taken by order of the government, and found to be 47,369. Seven years afterwards, the number of inhabitants was again taken; and found to be increaſed, by procreation only, above 14,000; and very near one *half* of the inhabitants were found

	Births.	Weddings.	Burials.
At Paris,	19,100	4,400	19,400
Vienna,	5,600		6,800
Amſterdam,	4,600	2,400	8,000
Copenhagen,	2,700	886	3,309
Berlin, for 5 years, ending at 1751	3,634	936	4,092

(*a*) See a diſcourſe on *Chriſtian Union*, by Dr. *Styles*, *Boſton*, 1761, p. 103. 109, &c.—See alſo *The Intereſt of Great Britain conſidered with regard to her Colonies, together with Obſervations concerning the increaſe of mankind, peopling of countries, &c.* p. 35. 2d edit. *London*, 1761.

to be under (*a*) 16 years of age. In 22 years, therefore, they muſt have doubled their own number, and the births muſt have exceeded the burials 2000 annually. As the increaſe here is much quicker than in *Madeira*, we may be ſure that a ſmaller proportion of the inhabitants muſt die annually. Let us, however, ſuppoſe it the ſame, or a 50th part. This will make the annual burials to have been, during theſe ſeven years, 1000; and the annual births 3000; or an 18th part of the inhabitants.—Similar obſervations may be made on the much quicker increaſe in *Rhode Iſland*, as related in the preface to the *Collection of the London Bills of Mortality*; and alſo in the valuable pamphlet, laſt quoted, on *the Intereſt of Great Britain with regard to her Colonies*, p. 36.—What a prodigious difference muſt there be, between the vigour and the happineſs of human life in ſuch ſituations, and in ſuch a place as *London?*—The original number of perſons who, in 1643, had ſettled in *New-England*, was 21,200. Ever ſince, it is reckoned, that more have left them than have gone to them (*b*). In the year 1760, they were increaſed to half a million. They have, therefore, all along

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(*a*) According to Dr. *Halley's* Table, the number of the living under 16, is but a *third* of all the living at all ages.

(*b*) See Dr. *Styles's* pamphlet juſt quoted, p. 110, &c.

doubled their own number in 25 years. And if they continue to increaſe at the ſame rate, they will, 70 years hence, in *New-England* alone, be four millions; and in all *North America*, above twice the number of inhabitants in *Great-Britain* (*a*).—But I am wandering

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(*a*) The rate of increaſe, ſuppoſing the procreative powers the ſame, depends on two cauſes: The "encouragement to marriage;" and the "*expectation* of a child juſt born." When one of theſe is given, the increaſe will be always in proportion to the other. That is; As much *greater* or *leſs* as the *ratio* is of the numbers who reach maturity, and of thoſe who marry, to the number born, ſo much *quicker* or *ſlower* will be the increaſe.—Let us ſuppoſe the operation of theſe cauſes ſuch, as to produce an annual exceſs of the *births* above the *burials*, equal to a 36th part of the whole number of inhabitants. It may ſeem to follow from hence, that the inhabitants would double their own number in 36 years; and thus ſome have calculated. But the truth is, that they would double their own number in much leſs time. Every addition to the number of inhabitants from the births, produces a proportionably greater number of births, and a greater exceſs of theſe above the burials; and if we ſuppoſe the exceſs to increaſe annually at the ſame rate with the inhabitants, or ſo as to preſerve the *ratio* of it to the number of inhabitants always the ſame, and call this *ratio* $\frac{1}{r}$, the period of doubling will be, the *quotient* produced by dividing the logarithm of 2, by the *difference* between the logarithms of $r + 1$ and r; as might be eaſily demonſtrated. In the preſent caſe, r being 36, and $r + 1$ being 37, the period of doubling comes out 25 years. If r is taken equal to 22, the period of doubling will be 15 years.—But it is certain that this ratio may, in many ſituations, be greater than $\frac{1}{22}$; and, inſtead of remaining the ſame, or becoming leſs, it may *increaſe*, the conſequence of which will be, that the period of doubling will be ſhorter than this rule gives it.—According to Dr. *Halley*'s Table, the number of perſons between 20 and

42

dering from my purpoſe in this letter. The point I had chiefly in view was, the preſent ſtate

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42 years of age is a third part of the whole number living at all ages. The prolific part, therefore, of a country may very well be a 4th of the whole number of inhabitants; and ſuppoſing four of theſe, or every other marriage between perſons all under 42, to produce *one* birth every year, the annual number of births will be a 16th part of the whole number of people. And, therefore, ſuppoſing the burials to be a 48th part, the annual exceſs of the births above the burials will be a 24th part, and the period of doubling 17 years. — The number of inhabitants in *New-England* was, as I have ſaid from Dr. *Styles's* pamphlet, half a million in 1760. If they have gone on increaſing at the ſame rate ever ſince, they muſt be now 640,000; and it ſeems to appear that in fact they are now more than this number. For, ſince writing the above obſervations, I have ſeen a particular account, grounded chiefly on ſurveys lately taken with a view to taxation and for other purpoſes, of the number of males, between 16 and 60, in the four provinces. According to this account, the number of ſuch males is 218,000. The whole number of people, therefore, between 16 and 60, muſt be nearly 436,000. In order to be more ſure of avoiding exceſs, I will call them only 400,000. In Dr. *Halley's* Table the proportion of all the living under 16 and above 60, to the reſt of the living, is 13.33 to 20; and this will make the number of people now living in the four provinces of *New-England* to be 666,000. But on account of the rapid increaſe, this proportion muſt be conſiderably greater in *New-England*, than that given by Dr. *Halley's* Table. In *New-Jerſey*, I have ſaid the number of people under 16, was found to be almoſt equal to the number above 16. Suppoſe, however, that in *New-England*, where the increaſe is ſlower, the proportion I have mentioned is only 16 to 20; and then the whole number of people will be 720,000.

I cannot conclude this note without adding a remark to remove an objection which may occur to ſome in reading Dr.

ſtate of *London* as to healthfulneſs, number of inhabitants, and its influence on population. The obſervations I have made may, perhaps, help to ſhew, how the moſt is to be made of the lights afforded by the *London* bills; and ſerve as a ſpecimen of the proper method of calculating from them. It is indeed extremely to be wiſhed, that they were leſs imperfect than they are, and extended further. More pariſhes round *London* might be taken into them; and, by an eaſy improvement in the pariſh regiſters now kept, they might be extended through all the pariſhes and towns in the kingdom. The advantages ariſing from hence would be very conſiderable. It would give the preciſe law according to which human life waſtes in its different ſtages; and thus ſupply the neceſſary *data* for computing accurately the values of all *life-annuities* and *reverſions*. It would, likewiſe, ſhew the different degrees of healthfulneſs

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Dr. *Heberden*'s account of *Madeira*, to which I have referred. In that account 5945 is given as the number of children under ſeven in the iſland, at the beginning of the year 1767. The medium of annual births, for eight years, had been 2201; of burials 1293. In ſix years, therefore, 13,266 muſt have been born; and if, at the end of ſix years, no more than 5945 of theſe were alive, 1210 muſt have died every year. That is; almoſt all the burials in the iſland, for ſix years, muſt have been burials of children under ſeven years of age. This is plainly incredible; and, therefore, it ſeems certain, that the number of children under ſeven years of age muſt, through ſome miſtake, be given, in that account, 3000 or 4000 too little.

ſulneſs of different ſituations, mark the progreſs of population from year to year, keep always in view the number of people in the kingdom, and, in many other reſpects, furniſh inſtruction of the greateſt importance to the ſtate. Mr. *De Moivre*, at the end of his book on the doctrine of chances, has recommended a general regulation of this kind; and obſerved, particularly, that at leaſt it is to be wiſhed, that an account was taken, at proper intervals, of all the living in the kingdom, with their ages and occupations; which would, in ſome degree, anſwer moſt of the purpoſes I have mentioned. — But, dear Sir, I am ſenſible it is high time to finiſh theſe remarks. I have been carried in them far beyond the limits I at firſt intended. I always think with pleaſure and gratitude of your friendſhip. The world owes to you many important diſcoveries; and your name muſt live as long as there is any knowledge of philoſophy among mankind. That your happineſs in this, and every other reſpect, may continually increaſe, is the ſincere wiſh of, SIR,

Your much obliged,

and very humble ſervant,

Newington-Green,
April 3, 1769.

RICHARD PRICE.

POSTSCRIPT.

AT *Edinburgh*, bills of mortality, of the ſame kind with thoſe in *London*, have been kept for many years. I have, ſince the foregoing letter was written, examined theſe bills, and formed a Table of Obſervations from them, as I found them for a period of 20 years, beginning in 1739, and ending in 1758.—As this is a town of moderate bulk, and ſeems to have a particular advantage of ſituation; I expected to find the probabilities of life in it, nearly the ſame with thoſe at *Breſlaw*, *Northampton* and *Norwich*; but I have been ſurprized to obſerve, that this is not the caſe. During the period I have mentioned, only one in 42 of all who died at *Edinburgh*, reached 80 years of age; which is a ſmaller proportion than attains to the ſame age in *London*. See p. 201.—In general; it appears, that the probabilities of life in this town are much the ſame, thro' all the ſtages of life, with thoſe in *London*, the chief difference being, that after 30, they are rather lower at *Edinburgh*.—It is not difficult to account for this. It affords, I think, a ſtriking proof of the pernicious effects ariſing from uncleanlineſs, and crouding together on one ſpot too many inhabitants. At *Edinburgh*, Mr. *Maitland* ſays, "the build-"ings, elſewhere called *houſes*, are denomi-

“ nated *lands*; and the *apartments*, in other “ places named *ſtories*, here called *houſes*, are “ ſo many freeholds inhabited by different “ families; whereby the houſes are ſo exceſſively crouded with people, that the “ inhabitants of this city may be juſtly preſumed to be more numerous than thoſe of “ ſome towns of *triple* its dimenſions.” See *Maitland's Hiſtory of Edinburgh*, p. 140.

In the year 1748, the whole number of *apartments* or *families* in the city and liberties of *Edinburgh*, was 9064. This Mr. *Maitland* mentions as the reſult of particular examination, and undoubtedly right. *Ib.* p. 217, 218.—In 1743, an accurate account was taken, by the deſire of this writer, of the number of *families* and *inhabitants* in the pariſh of St. *Cuthbert*. *Ib.* p. 171. The number of *families* was 2370, and of *inhabitants* at all ages, 9731. The proportion, therefore, of *inhabitants* to *families*, was $4\frac{1}{10}$ to 1; and, ſuppoſing this the true proportion for the whole town, the number of inhabitants will be $4\frac{1}{10}$ multiplied by 9064, or 37,162.—The yearly medium of deaths in the town and liberties for eight years, from 1741 to 1748, was 1783. *Ib.* p. 220 and 222. And, conſequently, *one* in $20\frac{4}{5}$ died annually.

Mr. *Maitland*, tho' poſſeſſed of the *data* from which theſe concluſions neceſſarily follow, has made the number of inhabitants 50,120, in conſequence of a diſpoſition to exaggerate in theſe matters, and of aſſuming, without any reaſon, a 28th part of the inhabitants as dying annually.

In page 220, he expreſſes much ſurprize at finding, that the number of males in this town was leſs than the number of females, in the proportion of 3 to 4. But this is by no means peculiar to *Edinburgh*.

All I have been ſaying muſt be underſtood of the ſtate of *Edinburgh*, before the year 1758. The bills, for the laſt 12 years, have been ſo irregular, and ſo different from the ſame bills for the preceding years, and from all other bills, that I cannot give them any credit. Either ſome particular incorrectneſs has crept into the method of keeping them; or there has been ſome change in the ſtate of the town which renders them of no uſe. Probably the former is the truth.

From the note in p. 203, it appears, that the chriſtenings and burials at PARIS, come very near to equality. This once led me to ſuſpect, that there muſt be ſome particular ſingularity in the ſtate of *Paris*, which rendered it much leſs prejudicial to health and population than great towns commonly are. But better information has lately obliged me to entertain very different ſentiments.—The difference between the births and burials at *Paris*, is much greater than the bills ſhew.

" Children here are baptized the inſtant " they are born; and, in a day or two af- " terwards, it is the cuſtom to ſend them to " the adjacent villages to be nurſed. A " great number therefore, of the infants born " at *Paris*, die in the country, and theſe " appear only in the regiſter of chriſten- " ings." See a book entitled the *Police of France*, page 127. And *Buffon*'s Natural Hiſtory, Tom. II. at the end.—" All the " children alſo received into the *Foundling-* " *Hoſpital*, are immediately ſent to be nurſ- " ed in the country, at a diſtance from *Paris*, " where they remain 5 or 6 years; at the end " of which time they are brought again to " *Paris*, the boys to be placed in the ſuburbs " of St. *Antoine*, and the girls at *Salpetriere*, " to be further maintained till they arrive at " the age of twelve years." *Police of France*, p. 81.—The following paſſage in the ſame writer, containing a further account of this Hoſpital, is important; and, therefore, tho' long, I cannot help tranſcribing it—" Let " us ſuppoſe, that out of 4000 children an- " nually carried into the country, two thirds " may die, during the five years they are de- " ſtined to remain at nurſe; ſo that only " 1333 would conſtantly be the annual " number ſent back to *Paris*; who, being " kept at the two Hoſpitals St. *Antoine* and " *Salpetriere* juſt mentioned, 'till they are 12, " and ſucceeded by a like number each year,

" the total number compoſed of all brought " in the ſucceſſive years, would make the " conſtant reſting ſtock to amount to 9331. " But of theſe we will ſuppoſe a 5th part " to die every year. Yet even then the " conſtant reſting ſtock of children ought to " be 7465. How greatly then muſt we be " ſurprized to find, by the authentic account " taken from their own books, only 640 " boys in the college of St. *Antoine*, and not " more tham 600 girls at the *Salpetriere*; " ſo that the reſting ſtock of returned foundlings appears to be no more than 1240, " which being deducted from 7465, will make " the difference in the deficiencies 6225. " What then becomes of theſe?—Are they " reclaimed by their parents?—Or do they " periſh for want of care?—In anſwer to " which queſtions it was explained to me; " that as many of the lower claſs of people " were induced to marry, in order to be excuſed from ſerving in the militia; ſo when " theſe have children, which they are unable to maintain, they uſually ſend them to " this hoſpital; which, therefore, muſt be " looked upon, as not only a charity for the " care of expoſed and deſerted children whoſe " parents are unknown, but alſo as a public " *nurſery* for the ſuſtenance of the children " of poor people, who, tho' regiſtered at the " office, are often reclaimed from their country nurſes by their parents. This accounts,

" in ſome meaſure, for the ſmall ſtock of " children brought back to the hoſpital at " *Paris*.—The further difference is ſuſpected " to be owing to the inſufficient nouriſhment " they receive; as this particular charity, as " well as the General Hoſpital, adopts that " prepoſterous method of taking in an un- " limited number, while there is only a li- " mited income for their ſubſiſtence." *Ib.* page 83.

Theſe facts prove, that, at the ſame time that the regiſter of *chriſtenings* at *Paris* muſt be full, the regiſter of *burials* muſt be very deficient. Let the deficiencies be reckoned at 4600; and, conſequently, the annual burials at 24,000. The annual average of weddings, given in p. 203, is 4300; and, therefore, the number of perſons who marry annually muſt be 8600. The difference between the chriſtenings and burials is 5000; which, therefore, is the number of annual recruits from the country. Theſe, in general, muſt be perſons in mature life. Suppoſe 3600 of them to marry after ſettling at *Paris*. Then, 8600 leſſened by 3600 or 5000, will be the number of perſons born at *Paris* who marry annually; and 14,100, or near three-fourths of all who are born at *Paris*, will be the number dying annually in childhood and celibacy.

The ſuppoſitions on which I have made this computation ſeem moderate; but if any one thinks otherwiſe, he may make the ſame calculation on any other ſuppoſitions.

The births at *Paris* are above four times the weddings; and it may ſeem, therefore, that here, as well as in the moſt healthy country ſituations, every wedding produces above four children. I have obſerved nothing like this in any other great town. Many children born in the country are, I ſuppoſe (*a*), brought to the Foundling-Hoſpital, and there chriſtened. This Hoſpital may likewiſe occaſion a more than common number of illegitimate births. And, beſides, ſome who leave the country to ſettle at *Paris*, may come thither already married. Theſe are circumſtances that will ſwell the regiſter of births, without having any effect on the weddings. I do not, however, know that any of them take place at *Paris*; and, perhaps, it muſt be granted, that it is diſtinguiſhed in this reſpect from moſt other towns. Nor can I wonder at this, if it be indeed true, not only, that all married men in *France* are excuſed ſerving in the militia from whence draughts are made for the army, but alſo, that a *fifth* of all the children born at *Paris* are ſent to the *Foundling-Hoſpital* (*b*). Theſe are

(*a*) "If the parents of a child brought to this Hoſpital "are known, the regiſter of its baptiſm muſt be pro- "duced. If the parents are unknown, the child muſt "be baptiſed after being received." *Police of France*, pag. 82.

(*b*) See Police of France, &c. p. 83.—This writer adds, that a *third* of all that die at *Paris* die in Hoſpitals.

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are encouragements to marriage that no other city enjoys. It has been ſeen that the *Foundling-Hoſpital*, tho' attended with this effect, is, probably, in the higheſt degree pernicious; but it is to be wiſhed, that ſome policy of the ſame kind with that *firſt* mentioned, was purſued in this kingdom.—At the end of the 2d vol. of Monſieur *De Buffon's Natural Hiſtory*, there are Tables formed from the Obſervations of M. *Du Pre de S. Mour*, of the *French Academy*, containing an account of the ages at which 13,189 perſons died in three pariſhes at *Paris*; and alſo, of the ages at which 10,805 perſons died in 12 country pariſhes and villages near *Paris*.—According to theſe Tables, many *more* die in the beginning of life, and much *fewer* in the latter part of life, in the country than in *Paris*. But the circumſtances of *Paris*, and the country round it, are ſuch, that no argument can be drawn

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" In the *Hotel Dieu* (a great Hoſpital ſituated in the " middle of the city) we may, he ſays, behold a horrid " ſcene of miſery; for, the beds being too few for the " numbers admitted, it is common to ſee 4, or 6, or " even 8 in a bed together, lying 4 at one end, and 4 " at the other, ill of various diſtempers in ſeveral de- " grees; ſome bad, others worſe; ſome dying, others " dead.—Above a *fifth* of all admitted to this Hoſpital " die; the annual numbers admitted being 21,823. The " medium of deaths for three years from 1751 to 1753, " 4650.—The medium of deaths for the ſame years in " *all* the Hoſpitals was 6181." *Ib.* p. 85.—In our two great city Hoſpitals, St. *Thomas*'s and St. *Bartholomew*'s, about 600 die annually; or one in 13 of all admitted as in-patients.

from hence in favour of *Paris*. Many of the children dying in the country, are children ſent thither from *Paris* to be nurſed; and, on the other hand, *many*, perhaps *moſt*, of thoſe who die in old age at *Paris*, are perſons who have removed thither from the country, ſome to *Hoſpitals*, and ſome to places and ſettlements. It is evident, therefore, that theſe Tables give a repreſentation of the probabilities of life at *Paris*, which, when compared with thoſe in the adjacent country (*a*), is juſt the reverſe of the truth. Were the children born at *Paris*, who die in the country, to be transferred to the town regiſter; and, on the contrary, the adults born in the country, who die at *Paris*, to be transferred to the country regiſter, there is no reaſon to doubt, but that the probabilities of life at *Paris*, would be found as low, in compariſon with thoſe in the country, as the probabilities of life in *London* are; or, perhaps, much lower. — This obſervation is applicable, in ſome degree, to moſt other great towns; and, in general, on account of the migrations from the country to towns, navies and armies, we may be ſatisfied, that we err on the ſide of *defect*,

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(*a*) It is for this reaſon that theſe Tables, when combined, exhibit juſtly the *mean* probabilities of life for town and country taken together; and that the Table of the *decrements* of life deduced from them by M. *Buffon* and M. *Du Pre*, agrees nearly with Dr. *Halley*'s Table.

whenever we judge of the probabilities of life in the *country*, from the numbers dying in the ſeveral ſtages of life; and, on the ſide of *exceſs*, whenever, in the ſame way, we judge of the probabilities of life in *towns*. And this, it is obvious, has a tendency to confirm all that has been ſaid in the preceding Eſſay, concerning the pernicious effects of great towns on human life.

There are ſeveral *ordonances* and *arrets* of council which fix the boundaries of *Paris*, and prohibit all new buildings beyond thoſe boundaries.—The reaſons of this regulation, as ſet forth in one of theſe *arrets*, are remarkable; and it will not be improper to recite them.—"By the exceſſive aggrandiz-"ing of the city, it is ſaid, the air would be "rendered unwholeſome, and the cleaning "the ſtreets more difficult."—"Augment-"ing the number of inhabitants would aug-"ment the price of proviſions, labour and "manufactures."—"That ground would be "covered with buildings, which ought to be "cultivated in raiſing the neceſſary ſubſiſt-"ence for the inhabitants; and thereby ha-"zard a ſcarcity."—"The people in the "neighbouring towns and villages would be "tempted to come and fix their reſidence in "the capital, and deſert the country."—"And laſtly; the difficulty of governing ſo "great a number of people would occaſion "a diſorder in the *Police*, and give an oppor-

" tunity to rogues to commit robberies and " murders (*a*)."

No one can think overgrown cities greater evils than I do. But, yet, I can by no means approve of this policy. The effect of it muſt be, crouding together too many people within the preſcribed boundaries, and rendering a town more the ſeat of uncleanlineſs, infection and diſeaſe.—The number of houſes in *Paris* is reckoned to be 28,000 (*b*), or 30,000; but the number of inhabitants, ſuppoſing a 20th part to die annually, cannot be much leſs than 480,000, or 16 times the number of houſes.

It is happy for LONDON, that there have been no laws to reſtrain its increaſe. In conſequence of being allowed to extend itſelf on all ſides into the country, the inhabitants now take near twice the room to live upon that they did; and it is become leſs the means of ſhortening human life. See p. 189, 191, and 201.

In page 203, I have given the annual *medium* of births, weddings and burials at BERLIN, from 1747 to 1751.—In 1747, an account was taken with the utmoſt care, by the order of the King of PRUSSIA, of the number of inhabitants in this town; and, it was found to be 107,224.—In order to be more certain, a *ſecond* account was taken the

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(*a*) Vid. Police of France, p. 130.

(*b*) *Ibid.*—There are other accounts which makes this number above 50,000.

ſame year; and the number found the ſame within 200.—In 1749, the inhabitants were increaſed to 110,933. Their number, therefore, compared with the annual burials, was as 27 to 1.—This is a higher proportion than could be expected in a town ſo conſiderable; and alſo ſo much crouded, as to have, at an average, 16 inhabitants in every houſe. But there is a plain reaſon to be given for this fact. — BERLIN, for many years, had been increaſing very faſt, by a conflux of people from the ſurrounding country and provinces. About the year 1700, the medium of annual burials was no more than 1000. In 50 years, therefore, it quadrupled itſelf.—In a city increaſing with ſuch rapidity, the *ratio* of inhabitants to the annual deaths, muſt be greatly below the juſt ſtandard.—Were there now, ſuch acceſſions to LONDON of deſerters from the country, in the beginning of mature life, as would cauſe the number of inhabitants to increaſe at the rate of 10,000 every year, it would in 60 years be doubled; and the proportion of inhabitants to deaths would riſe gradually, 'till it came to be about one third greater. BERLIN, we have ſeen, has, in fact, increaſed at *more* than *double* this rate; and, therefore, the proportion of inhabitants dying annually in it is in reality very low.

The ingenious *Suſmilch*, to whoſe curious book, already quoted, I owe my information concerning BERLIN, makes the proportion of people who die annually in *great* towns, to be from $\frac{1}{24}$ to $\frac{1}{28}$; in *moderate* towns, from $\frac{1}{28}$ to $\frac{1}{31}$; and, in the country from $\frac{1}{40}$ to $\frac{1}{50}$. —The obſervations and facts in this Eſſay, joined to thoſe which will be found in the 4th Eſſay, prove, I think, that theſe proportions may be more truly ſtated as follows.— *Great* towns, from $\frac{1}{20}$ to $\frac{1}{23}$ or $\frac{1}{24}$. *Moderate* towns, from $\frac{1}{23}$ to $\frac{1}{28}$. The *country*, from $\frac{1}{30}$ or $\frac{1}{35}$, to $\frac{1}{50}$ or $\frac{1}{60}$.—This, however, muſt be underſtood with exceptions. There may be *moderate* towns ſo ill ſituated, or whoſe inhabitants may be ſo crouded together, as to render the proportion of deaths in them greater than in the largeſt towns: And, of this, EDINBURGH, if it is not now, was 20 years ago, an example.—There may be alſo *great* towns in which, from a ſudden increaſe, this proportion may be leſs than in ſmall towns: And of this, I have juſt given an example in BERLIN.

ESSAY II.

On Mr. De Moivre's *Rules for calculating the Values of* Joint *Lives.*

THE calculation of the values of *ſingle* and *joint* lives, from given Tables of Obſervation, being tedious and troubleſome; Mr. *De Moivre* has had recourſe to two *Hypotheſes*, which give eaſy rules for this purpoſe; and which, he thought, correſponded with ſufficient exactneſs to Obſervations.— The firſt of theſe *Hypotheſes* is, that the probabilities of life decreaſe, as we advance from childhood to old age, in an *arithmetical progreſſion*; or in ſuch a manner, that the *difference* is always the ſame, between the number of perſons living at the beginning of any one year, and the number living at the beginning of the next following year. — The other *Hypotheſis* is, that the probabilities of life decreaſe in a *geometrical* progreſſion; or in ſuch a manner, that the *proportion* is always the ſame, between the number of perſons living at the beginning of any one year, and the number living at the beginning of the next following year. — All the Tables of Obſervation ſhew, that the real law, according to which human life waſtes, comes much nearer to the former *Hypotheſis*, than the latter.—In Tables III, IV, and V, in the *Appendix*, it is ſo near the former *Hypotheſis*, that the difference is ſcarcely worth regarding. According to this *Hypotheſis*, therefore, (accommodated to the *Breſlaw* Table, in the manner mentioned in the note, page 2.) Mr. *De Moivre* calculated the values of *ſingle* lives; and the rules founded upon it for this purpoſe are ſo eaſy, that an operation which would otherwiſe take up much time, may be performed almoſt immediately.

By proceeding on the ſame principles, the values of *joint* lives might have been calculated; but the rules for this purpoſe derived from theſe principles, are far from being equally eaſy in practiſe. Here, therefore, Mr. *De Moivre* quitted his *firſt* Hypotheſis; and finding, that the *ſecond* Hypotheſis afforded, in the caſe of *joint* lives, rules that were as eaſy, as the rules given by the other Hypotheſis were in the caſe of *ſingle* lives, he choſe to adopt this *Hypotheſis*; believing at the ſame time, that the values of *joint* lives, obtained by rules derived from it, would not deviate much from the truth. But in this he was greatly miſtaken. The values of *two joint* lives obtained by theſe rules are ſo wrong, that in finding the preſent value, in a *ſingle payment*, of one life after another, they generally give reſults which are near $\frac{1}{4}$ of the true value too great; and about *two-fifths*

fifths too great, when the value is ſought in *annual payments* during the joint lives.— Theſe are errors ſo conſiderable, that I think it is of particular importance that the public ſhould be informed of them, in order to prevent the inconveniencies and perplexities they may occaſion.

Mr. *Simpſon* (in the Appendix to his Treatiſe on the *Doctrine of Annuities and Reverſions*) has obſerved, that Mr. *De Moivre*'s rules for finding the values of joint lives are wrong. But I don't know, that it has been ever attended to, that they are *ſo* wrong as I have found them. Mr. *Simpſon*'s remarks point out chiefly the errors in theſe rules, when the values of *three* or more joint lives are calculated by them; but, 'till I was forced to a particular examination of this ſubject by ſome difficulties into which I found myſelf brought by following Mr. *De Moivre* too implicitly, I did not at all ſuſpect, that any ſuch errors as I have mentioned, could ariſe from theſe rules, when the values of only *two* joint lives are calculated by them. Mr. *De Moivre*, in conſequence of other remarks contained in Mr. *Simpſon*'s *Appendix*, altered in the 4th edition of his Treatiſe ſome of his rules. It is ſurprizing he did not ſee reaſon at the ſame time to alter theſe.

That there may be no doubt about the truth of theſe obſervations, I will juſt mention a few examples of the difference between the values of a given reverſionary annuity, according to the rules to which I have objected, and the *true* values, according to the exact method of deducing them from Mr. *De Moivre's firſt Hypotheſis.*

Let the propoſed annuity be 30*l.*, to be enjoyed for what ſhall happen to remain of the life of a perſon now 40 years of age, after the life of another perſon of the ſame age. The value of the joint lives (intereſt being at 4 *per cent.*) is, by problem 2d of Mr. *De Moivre's* Treatiſe on Life-annuities, 8.964; which ſubtracted from 13.196, (the value by Table VI, of a ſingle life at 40) gives 4.23; which remainder, multiplied by 30, gives *l.*126.9, or the value of the reverſion in a ſingle preſent payment. And 126.9, divided by the foregoing value of the joint lives, is *l.* 14. 16; or, the value of the reverſion in annual payments during the joint lives.—But the *true* values are *l.*101.1 in a *ſingle* payment, by Queſt. I. chap. I.; and *l.* 10.3, in *annual* payments, by Queſt. IV.—The former values, therefore, are near a *quarter* of the true value too great in the *ſingle* payment; and near *two-fifths* too great in the *annual* payments.

The *true* value of the ſame annuity for a life at 66, after another life of the ſame age, is, (reckoning intereſt as before, at 4 *per cent.*) 68*l.* in a *ſingle* payment; and 13.5, in *annual* payments.—But theſe values, according to the Problem juſt quoted, are 91*l.*, and 21*l.* one of which is near a *third*, and the other above *half* the true value too great.

In *unequal* lives theſe errors may be no leſs conſiderable.—Thus; if the value of the propoſed annuity be required for a life at 70, after a life at 30 years of age; it will, by the ſame Problem, be *l.* 26.5, in a *ſingle* payment; and *l.* 5.1, in *annual* payments during the joint lives. But the *true* values are 17*l.* and *l.* 3.05.

Where 3 or more lives are concerned the errors will be ſtill greater.

The true values of the joint lives, mentioned in theſe Examples, have been calculated by a rule in pag. 16, of Mr. *Simpſon's* Treatiſe on the Doctrine of *Annuities* and *Reverſions*, and explained in note (M) *Appendix*.—To ſave, however, a great deal of trouble hereafter, I have thought proper to calculate Table VII, which gives the exact values, according to Mr. *De Moivre's firſt* Hypotheſis, of two joint lives, for every five years of human life from, 10 to 70.

This *Hypotheſis*, I have obſerved, does not differ much from the Tables of Obſervation in the *Appendix*, for *Breſlaw*, *Northampton* and *Norwich*. Between the ages of 30 and 40, it gives the values of *ſingle* lives almoſt the ſame with the *Breſlaw* Table. Under 30, it gives them ſomewhat *leſs*; and above 40, ſomewhat *greater*. But it ought to be remembered,

membered, that wherever it does this, it gives, at the ſame ages, the values of the *joint* lives alſo too little or too great; and that, conſequently, the reſults from it, in calculating the values of *Reverſions*, and of the *longeſt* of given lives, come ſo much nearer to exactneſs.

The rules to which I have objected are the only ones given by Mr. *De Moivre*, in all the editions of his Treatiſe on Life-Annuities. But it ſeems, this great mathematician became at laſt ſenſible, that they were too incorrect; and, therefore, at the end of the laſt edition of his Treatiſe on the *Doctrine of Chances*, pag. 320, (a work which gets into comparatively few hands) he has given other rules which come nearer the truth. But even theſe rules produce errors ſo great in many caſes, (particularly when combined with the errors of the Hypotheſis) that it will be beſt never to uſe them.

ESSAY III.

Of the Method of calculating the Values of Reverſions depending on Survivorſhips.

ALL Queſtions relating to the values of lives and reverſions, are at preſent of particular importance in this kingdom. Much buſineſs is continually tranſacted in this way; and any conſiderable errors in the methods of ſolving ſuch queſtions, muſt in time produce very bad conſequences.—The deſign of the following obſervations is to point out a particular error, into which there is danger of falling, in finding the values of ſuch reverſions as depend on ſurvivorſhips. In doing this, I ſhall, in order to be as plain as poſſible, take the following caſe. "A, aged "40, expects to come to the poſſeſſion of "an eſtate, ſhould he ſurvive B, aged like- "wiſe 40. In theſe circumſtances he offers, "in order to raiſe a preſent ſum, to give ſe- "curity for 40*l. per annum*, out of the eſtate "at his death, provided he ſhould get into "poſſeſſion; that is, provided he ſhould ſurvive B. What is the ſum that ought now "to be advanced to him, in conſideration of "ſuch ſecurity, reckoning compound intereſt "at 4 *per cent?*"

Mr. *De Moivre*'s directions in his Treatiſe on Annuities, Problems 17th and 20th, lead us to ſeek the required ſum in this caſe, by the following proceſs.

Find firſt, the preſent ſum A ſhould receive, for the reverſion of 40*l.* *per annum* for ever after his death; ſuppoſing it *not* dependent on his ſurviving B. The preſent value of ſuch a reverſion is "the (*a*) value of the life "ſubtracted from the *perpetuity*, and the *re-* "*mainder* multiplied by the annual rent."— The value of the life is, by Table VI, 13.196. This ſubtracted from 25, the *perpetuity*, leaves 11.80; which, multiplied by 40, gives *l.*472; the value of the ſuppoſed eſtate, after the life of A. But, as Mr. *De Moivre* obſerves, the lender having a chance to loſe his money, a compenſation ought to be made to him for the riſk he runs, which is founded on the poſſibility, that a man of 40 years of age may not ſurvive another perſon of the ſame age. This chance is an *equal* chance; and, therefore, half the preceding ſum, or 236*l.* is the money which ſhould be advanced now on the expectation mentioned.

(*a*) By *Scholium*, p. 34, and Problem 26th, p. 293, of Mr. *Simpſon*'s Select Exerciſes.

This ſolution carries a plauſible appearance; and moſt perſons will, probably, be ready to pronounce it right; nor will this be at all wonderful, as ſo great a maſter of theſe ſubjects as Mr. *De Moivre* appears to have been miſled by it. — Nothing more is neceſſary to prove it to be fallacious, than proceeding in the ſame way to ſolve the following ſimilar Queſtion.

" A, aged 40, offers to give ſecurity for " 40*l. per annum*, to be entered upon at his " death, provided it ſhould happen *before* the " death of B, aged likewiſe 40. What ſum " ſhould now be advanced to him for ſuch a " reverſion, intereſt being reckoned at 4 *per* " *cent?*"

In ſolving this Problem, agreeably to the method juſt deſcribed, we are to find the value of 40*l. per annum*, to be entered upon *certainly* at the death of A; and then to multiply this value by the chance that A ſhall *not* ſurvive B, or by $\frac{1}{2}$; and in this way the anſwer comes out the ſame with that already given.

Now it may be eaſily ſeen, that this muſt be wrong. The value of a reverſion, to be received when a perſon of a given age dies, cannot be the ſame, whether the condition of obtaining it is, that he ſhall die *before*, or that he ſhall die *after* another perſon. That is, whether it is provided, that a purchaſer, if he ſucceeds, ſhall get into poſſeſſion *ſooner* or *later*. The reverſion in the latter caſe muſt, without doubt, be of leſs value than in the former.

The firſt Queſtion here propoſed, reſolves itſelf into the following general Queſtion.

"What is the preſent value of a given re-
"verſionary eſtate, to be entered upon after
"the failure of two lives, provided one *in*
"*particular* of them ſhould be the *longeſt*
"*life?*"

Now, the preſent value of an eſtate to be enjoyed for ever, after the failure of the *longeſt* of two lives, is "the value of the *longeſt* "of the two lives, ſubtracted from the *perpetuity*; and the remainder multiplied by "the annual rent of the eſtate."—The value of the *longeſt* of two lives is, (as is well-known) the value of the two *joint* lives, ſubtracted from the *ſum* of the (*a*) values of the two *ſingle* lives. In the preſent caſe, therefore, it is 9.82, (the value of two joint lives at the age of 40 by Table VII,) ſubtracted from twice 13.196; (the value of a *ſingle* life at the ſame age by Table VI,) that is, 16.57 year's purchaſe. And this ſubtracted from 25, (the perpetuity) gives 8.43; which, multiplied by 40, gives *l*.337.2, the value of the given eſtate were it *certainly* to be enjoyed, after the ex-

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(*a*) See Mr. *De Moivre* on Annuities, Problem IV; or Mr. *Simpſon's Doctrine of Annuities and Reverſions*, Prob. II.

tinction of the longeſt of two lives both 40; that is, whether *one* or *other* of them failed laſt. But that A's life in particular ſhould fail laſt, rather than B's, is an even chance. The true value of the reverſion, therefore, is half the laſt value, or *l.* 168.6.

In like manner. The ſecond Queſtion is the ſame with the Queſtion, "What is the preſent value of 40*l. per ann.* for ever, to be entered upon after the extinction of two *joint* lives both 40; that is, whenever *either* of them ſhall fail; provided the firſt that fails ſhould happen to be A's life in particular?" —And the anſwer is found by ſubtracting the preſent value of the *two joint* lives from the *perpetuity*, and multiplying the remainder by $\frac{1}{2}$, or by the chance that A in particular ſhall die firſt: And this will give the required value, *l.* 303.4 (*a*).

In ſhort. It appears in *both* theſe caſes, that, according to the firſt method of ſolution, we are to ſubtract from the *perpetuity* the value of *one* of the ſingle lives; when, in the *former* caſe, the value of the *longeſt* of the two lives; and, in the *latter* caſe, the value of their *joint continuance*, ought, in reality, to be ſubtracted. I need not ſay what prodigious errors may often ariſe from hence; and how unfit ſuch a method of ſolution is for practice.

(*a*) I have, tho' ſcarcely neceſſary, given a demonſtration of theſe Solutions in the Appendix, note (N).

Mr. *Simpſon*, in p. 322, of his Select Exerciſes, ſpeaks on this ſubject in the following manner.—"I have been very particular
"on theſe kinds of Problems; and the more
"ſo, as there has been no method before pub-
"liſhed, that I know of, by which they can
"be rightly determined. 'Tis true, the man-
"ner of proceeding, by firſt finding the pro-
"bability of ſurvivorſhip, (which method is
"uſed in my former work, and which a cele-
"brated author has largely inſiſted on in three
"ſucceſſive editions) may be applied to good
"advantage, when the given ages are nearly
"equal; but then it is certain, that this is
"not a genuine way of going to work, and
"that the concluſions hence derived are at
"beſt but near approximations."

This excellent mathematician has here expreſſed himſelf much too favourably of the method of ſolution on which I have remarked.—In both the caſes I have ſpecified the ages are equal; and yet, in one of them the error is a good deal above a *third* of the true value, and in the other, a *fifth:* And, it is obvious, that in caſes where three equal lives are taken, the errors will be much greater.—Mr. *Simpſon*'s Obſervations in this paſſage are true only, when applied to a *different* method uſed by himſelf, in the 28th and following Problems of his Treatiſe on the *Doctrine* of *Annuities* and *Reverſions*. This method is exact when the lives are equal; but, it gives reſults which are too far from the truth, when there is any conſiderable inequality between the lives.

It is with reluctance I have made ſome of theſe remarks. Mr. *De Moivre* has made very important improvements in this branch of ſcience; and the higheſt reſpect is due to his name and authority. This, however, only renders theſe remarks more neceſſary.

In the firſt Chapter (Queſtions 10th, 11th, 12th, 14th, &c.) I have given a minute account of the method of finding, in all caſes, the values of the reverſions which have been the ſubject of this Eſſay.

Obſervations on the proper Method of conſtructing Tables for determining the Rate of human Mortality, the Number of Inhabitants, and the Values of Lives in any Town or Diſtrict, from Bills of Mortality in which are given, the Numbers dying annually at all Ages.

IN every place that juſt ſupports itſelf in the number of its inhabitants, without any recruits from other places; or where, for a courſe of years, there has been no increaſe or decreaſe, the number of perſons dying every year at any particular age, and above it, muſt be equal to the number of the living at that age.—The number, for example, dying every year, at all ages, from the *beginning* to the utmoſt *extremity* of life, muſt, in ſuch a ſituation, be juſt equal to the whole number *born* every year. And for the ſame reaſon, the number dying every year at *one* year of age and upwards; at *two* years of age and upwards; at *three* and upwards, and ſo on; muſt be equal to the numbers that attain to thoſe ages every year; or, which is the ſame, to the numbers of the living at thoſe ages. It is obvious, that unleſs this happens, the number of inhabitants cannot remain the ſame. If the former number is greater than the latter, the inhabitants muſt *decreaſe*; if leſs, they muſt *increaſe*.—From this obſervation it follows, that in a town or country, where there is no increaſe or decreaſe, bills of mortality which give the ages at which all die, will ſhew the exact number of inhabitants; and alſo the exact law, according to which human life waſtes in that town or country.

In order to find the number of inhabitants; the mean numbers dying annually, at every particular age and upwards, muſt be taken as given by the bills, and placed under one another in the order of the ſecond column of the 12th Table in the Appendix. Theſe numbers will, it has appeared, be the numbers of the living at 1, 2, 3, &c. years of age; and, conſequently, the ſum, diminiſhed by half the number born annually (*a*), will be the whole

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(*a*) This ſubtraction is neceſſary for the following reaſon.—In a Table formed in the manner here directed, it is ſuppoſed, that the numbers in the ſecond column are all living together at the beginning of every year. Thus; the number in the *ſecond* column oppoſite to 0 in the *firſt* column, the Table ſuppoſes to be all juſt born together on the firſt day of the year. The number, likewiſe, oppoſite to 1, it ſuppoſes to attain to one year of age whole number of inhabitants.—In ſuch a ſeries of numbers, the exceſs of each number above that which immediately follows it, will be the number dying every year, out of the particular number alive at the beginning of the year; and theſe exceſſes ſet down regularly as in the third column of the Table to which I have referred, will ſhew the different rates at which human life waſtes thro' all its different periods, and the different probabilities of life at all particular ages.

It muſt be remembered, that what has been now ſaid goes on the ſuppoſition, that the place whoſe bills of mortality are given, ſupports itſelf, by procreation only, in the number of its inhabitants. In towns this very ſeldom happens, on account of the luxury and debauchery which generally prevail in them. They are, therefore, commonly kept up by a conſtant acceſſion of ſtrangers or *ſettlers*, who

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age juſt at the ſame time that the former number is born. And the like is true of every number in the ſecond column.—During the courſe of the year, as many will die at all ages as were born at the beginning of the year; and, conſequently, there will be an exceſs of the number alive at the beginning of the year, above the number alive at the end of the year, equal to the whole number of the annual births; and the true number conſtantly alive together, is the arithmetical mean between theſe two numbers; or, agreeably to the rule I have given, the *ſum* of the numbers in the ſecond column of the Table, leſſened by *half* the number of annual births. See Eſſay I, page 174.

who remove to them from country pariſhes and villages. In theſe circumſtances, in order to find the true number of inhabitants, and probabilities of life, from bills of mortality containing an account of the ages at which all die; it is neceſſary, that the proportion of the annual births to the annual ſettlers ſhould be known; and alſo the period of life at which the latter remove.—Both theſe particulars may be diſcovered in the following method.

If for a courſe of years there has been no ſenſible increaſe or decreaſe in a place, the number of annual ſettlers will be equal to the exceſs of the annual burials above the annual births. If there is an *increaſe*, it will be *greater* than this exceſs. If there is a *decreaſe*, it will be *leſs*.

The period of life at which theſe ſettlers remove, will appear in the bills by an increaſe in the number of deaths at that period and beyond it. Thus; in the *London* bills, the number of deaths, between 20 and 30, is generally above double; and between 30 and 40, near triple the number of deaths between 10 and 20: And the true account of this is, that from the age of 18 or 20, to 35 or 40, there is an afflux of people every year to *London* from the country, which occaſions a great increaſe in the number of inhabitants at theſe ages; and, conſequently, raiſes the deaths for all ages *above* 20, conſiderably above their due proportion, when compared with the number of deaths *before* 20. — This is obſervable in all the bills of mortality for towns with which I am acquainted, not excepting even the *Breſlaw* bills. Dr. *Halley* takes notice, that theſe bills gave the number of deaths, between 10 and 20, too ſmall. This he conſidered as an irregularity in them, owing to chance; and, therefore, in forming his Table of Obſervations, he took the liberty ſo far to correct it, as to render the proportion of thoſe who die to the living in this diviſion of life, nearly the ſame with the proportion which, he ſays, he had been *informed* (*a*) die annually of the young lads in *Chriſt-Church Hoſpital*. But the truth is, that this irregularity in the bills was derived from the cauſe I have juſt aſſigned.—During the five years for which the *Breſlaw* bills are given by Dr. *Halley*, the births did, indeed, a little exceed the burials; but, it appears, that this was the effect of ſome peculiar cauſes that happened to operate juſt at that time; for, during a complete century from 1633 to 1734, the annual medium of *births* was 1089 (*b*), and of burials

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(*a*) See *Lowthorp*'s Abridgment of the Philoſophical Tranſactions, vol. III. p. 670.—Dr. *Halley*'s information in this inſtance was not right, as will appear preſently; and, therefore, he has by no means ſufficiently corrected the irregularity I have mentioned.

(*b*) See Dr. *Short*'s Comparative Hiſtory, p. 63.

rials 1256 (*a*). This town, therefore, muſt have been all along kept up by a number of yearly recruits from other places, equal to about a *ſeventh* part of the yearly births.

What has been now obſerved, concerning the period of life at which people remove from the country to ſettle in towns, would appear ſufficiently probable, were there no ſuch evidence for it as I have mentioned; for it might be well reckoned, that theſe people in general, muſt be ſingle perſons in the beginning of mature life, who, not having yet obtained ſettlements in the places where they were born, migrate to towns in queſt of employments.

Having premiſed theſe Obſervations, I ſhall next endeavour to explain diſtinctly, the effect which theſe acceſſions to towns muſt have, on Tables of Obſervation formed from their bills of mortality. This is a ſubject proper to be inſiſted on, becauſe miſtakes have been committed about it; and becauſe alſo, the diſcuſſion of it is neceſſary to ſhew, how near to truth the values of lives come as deduced from ſuch Tables.

The following general rule may be given on this ſubject.

(*a*) It appears from the account in the *Philoſophical Tranſactions*, (Abridgment, vol. VII, No. 380, p. 46, &c.) that from 1717 to 1725, the annual medium of births at *Breſlaw* was 1252, of burials 1507; and alſo, that much the greateſt part of the births died under 10 years of age.

If a place has, for a courſe of years, been maintained in a ſtate nearly ſtationary, as to number of inhabitants, by recruits coming in every year, to prevent the decreaſe that would ariſe from the exceſs of the burials above the births; a Table formed on the principle, "that the number dying annually, after every "particular age, is equal to the number living at that age," will give the number of inhabitants, and the probabilities of life, too *great*, for all ages preceding that at which the recruits ceaſe; and after this, it will give them *right*.—If the acceſſions are ſo great as to cauſe an *increaſe* in the place, ſuch a Table will give the number of inhabitants, and the probabilities of life, too *little*, after the age at which the acceſſions ceaſe (*a*); and too great, if there is a decreaſe. *Before* that age it will in *both* caſes give them too great; but moſt conſiderably ſo in the former caſe, or when there is an increaſe.

(*a*) Agreeably to theſe Obſervations; if a place increaſes, not in conſequence of acceſſions from other places, but of a conſtant exceſs of the births above the deaths; a Table, conſtructed on the principle I have mentioned, will give the probabilities of life too low through the *whole extent* of life; becauſe, in ſuch circumſtances, the number of *deaths* in the *firſt* ſtages of life muſt be too great, in compariſon of the number of deaths in the *latter* ſtages; and more or leſs ſo, as the *increaſe* is more or leſs rapid.—The contrary, in all reſpects, takes place where there is a decreaſe, ariſing from the exceſs of the *deaths* above the *births*.

For example. Let us ſuppoſe, that 244 of thoſe born in a town, attain annually to 20 years of age; and that 250 more, all likewiſe 20 years of age, come into it annually from other places; in conſequence of which, it has, for a courſe of years, been juſt maintained in the number of its inhabitants, without any ſenſible increaſe or decreaſe. In theſe circumſtances, the number of the living in the town of the age of 20, will be always 244 *natives* and 250 *ſettlers*, or 494 in all; and, ſince theſe are ſuppoſed all to die in the town, and no more recruits are ſuppoſed to come in; 494 will be likewiſe the number dying annually at 20 and upwards.—In the ſame manner; it will appear on theſe ſuppoſitions, that the number of the living, at every age, ſubſequent to 20, will be equal to the number dying annually at that age and above it; and, conſequently, that the number of inhabitants and the decrements of life, for every ſuch age, will be given exactly by the Table I have ſuppoſed. But for all ages *before* 20, they will be given much too great. For let 280 of all born in the town, reach 10. In this caſe, 280 will be the true number of the living in the town at the age of 10; and the recruits not coming in 'till 20, the number given by the bills, as dying between 10 and 20, will be the true number dying annually of the living in this diviſion of life. Let this number be 36; and it will follow, that the Table ought to make the numbers of the living at the ages between 10 and 20, a ſeries of decreaſing means between 280 and (280 diminiſhed by 36, or) 244. But in forming the Table on the principle I have mentioned, 250 (the number above 20 dying annually in the town who were not born in it) will be added to each number in this ſeries; and, therefore, the Table will give the numbers of the living, and the probabilities of life in this diviſion of life, almoſt twice as great as they really are.—This obſervation, it is manifeſt, may be applied to *all* the ages under 20.

It is neceſſary to add, that ſuch a Table will give the number of inhabitants, and the probabilities of life, equally wrong before 20, whether the recruits all come in at 20, agreeably to the ſuppoſition juſt made, or only *begin* then to come in. In this laſt caſe, the Table will give the number of inhabitants, and probabilities of life, too great throughout the whole extent of life, if the recruits come in at all ages above 20. But if they ceaſe at any particular age, it will give them right only from that age; and before, it will err all along on the ſide of exceſs; but leſs conſiderably between 20 and that age, than before 20. ——For example. If, of the 250 I have ſuppoſed to come in at 20, only 150 then come in, and the reſt at 30; the numbers of the living will be given 100 too high, at every age between 20 and 30; but, as juſt ſhewn, they will be given 250 too high at every age before 20.—In general, therefore, the number of the living *at* any particular age, muſt be given by the ſuppoſed Table, as many too great as there are annual ſettlers *after* that age; and, if theſe ſettlers come in at all ages indiſcriminately, during any certain interval of life; the number of inhabitants and the probabilities of life will be continually growing leſs and leſs wrong, the nearer any age is to the end of that interval. —Theſe obſervations prove, that Tables of Obſervation formed in the common way, from bills of mortality for places, where there is an exceſs of the burials above the births, muſt be erroneous, for a great part of the duration of life, in proportion to the degree of that exceſs. They ſhew likewiſe, at what parts of life the errors in ſuch Tables are moſt conſiderable, and how they may be in a great meaſure corrected.

All this I ſhall beg leave to exemplify and illuſtrate a little further, in the particular caſe of *London*.

The number of deaths, between the ages of 10 and 20, is always ſo ſmall in the *London* bills, that it ſeems certain few recruits come to *London* under 20; or at leaſt, not ſo many as before this age are ſent out for education to ſchools and univerſities. After 20, great numbers come in 'till 30, and ſome, perhaps, 'till 40 or 50. — But, at every age after 50, it is probable, that more retire from *London* than come to it.—The *London* Tables of Obſervation, therefore, being formed on the principle I have mentioned, cannot give the probabilities of life right 'till 40. Between 30 and 40 they muſt be a little too high; but more ſo between 20 and 30; and moſt of all ſo before 20.—It follows alſo, that theſe Tables muſt give the number of inhabitants in *London* much too great.

Table XII, in the Appendix, is a Table formed in the manner I have explained, from the *London* bills for 10 years, from 1759 to 1768; and adapted to a 1000 born as a *radix*. The ſum of the numbers in the ſecond column, diminiſhed by half the number born, is 25,757. According to this Table then, for every 1000 deaths in *London*, there are 25¾ as many inhabitants; or, in other words, the expectation of a child juſt born is 25¾; and the inhabitants are to the annual burials, as 25¾ to 1. —But it has appeared, that the numbers in the ſecond column being given on the ſuppoſition, that all who die in *London* were born there, muſt be too great; and we have from hence a DEMONSTRATION, that the probabilities of life are given in the common Tables of *London* Obſervations, too high, for, at leaſt, the firſt 30 years of life; and alſo, that the number of inhabitants in *London* muſt be *leſs*, than 25¾, multiplied by the annual burials. —The common Tables, therefore, of *London*

Obſervations, undoubtedly want to be corrected (*a*); and the way of doing this, and in general, the right method of forming genuine Tables of Obſervation for towns, may be learnt from the following rule.

"From the ſum of all that die annually, "*after* any given age, ſubtract the number "of annual ſettlers *after* that age; and the "remainder will be the number of the liv- "ing *at* the given age."

This rule can want no explication or proof, after what has been already ſaid.

If, therefore, the number of annual ſettlers in a town at every age could be aſcertained; a perfect Table of Obſervations might be formed for that town, from bills of mortality, containing an account of the ages at which all die in it. But no more can be learnt in this inſtance, from any bills, than the whole number of annual *ſettlers*, and the general diviſion of life in which they enter. This, however, may be ſufficient to enable us to form Tables that ſhall be tolerably exact.—For inſtance. Suppoſe the *annual deaths* in a town which has not increaſed or decreaſed,

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(*a*) The ingenious and accurate Mr. *Simpſon* ſaw that it was neceſſary to correct the *London* Tables, and he has done it with great judgment; but, I think, too imperfectly, and without going upon any fixt principles, or ſhewing particularly, how Tables of Obſervation ought to be formed, and how far in different circumſtances, and at different ages, they are to be depended on.

to have been for many years, in the proportion of 4 to 3, to the *annual births*. It will hence follow, that $\frac{1}{4}$ of the perſons who die in ſuch a town are *ſettlers*, or *emigrants* from other places; and not *natives:* And the ſudden increaſe in the deaths after 20, will alſo ſhew, agreeably to what was before obſerved, that they enter after this age. In forming, therefore, a Table for ſuch a town, a *quarter* of all that die at all ages throughout the whole extent of life, muſt be deducted from the ſum of all that die after every given age before 20; and the remainder will be the true number living at that given age. And if, at 20, and every age above it, this deduction is omitted, or the number of the living at every ſuch age is taken the ſame with the ſum of all that die after it, the reſult will be (ſuppoſing *moſt* of the ſettlers to come in before 30, and *all* before 40) a Table exact 'till 20; too high between 20 and 30; but nearly right for ſome years before 40; and after 40 exact again.—Such a Table, it is evident, will be the ſame with the Table laſt deſcribed at all ages above 20; and different from it only under 20.—It is evident alſo that, on account of its giving the probabilities of life too great for ſome years after 20, the number of inhabitants deduced from it may be depended on as ſomewhat greater than the truth; and more or leſs ſo, as the annual recruits enter in general later or ſooner after 20.

Let us now conſider, what the reſult of theſe remarks will be, when applied particularly to the *London* bills.

It muſt be here firſt obſerved, that, at leaſt, one *quarter* of all that die in *London* are *ſettlers* from the country, and not *natives*.—The medium of annual burials for 10 years, from 1759 to 1768, was 22,956; of births 15,710. The exceſs is 7246; or near a *third* of the burials.—The ſame exceſs, during 10 years, before 1750, was 10,500; or, near *half* the burials. *London* was then *decreaſing*. For the laſt 12 or 15 years it has been *increaſing*. This exceſs, therefore, agreeably to the foregoing obſervations, was then *greater* than the number of annual ſettlers; and it is now *leſs*. I have choſen, however, to ſuppoſe the number of annual ſettlers to be now, no more than a *quarter* of the annual burials, in order to allow for more omiſſions in the births than the burials; and alſo, in order to be more ſure of obtaining reſults that ſhall not exceed the truth.

Of every thouſand then who die in *London*, only 750 are *natives*, and 250 are *ſettlers*, who come to it after 18 or 20 years of age: And, conſequently, in order to obtain from the bills a more correct Table than the 12th in the *Appendix*, 250 muſt be ſubtracted from every one of the numbers in the ſecond column 'till 20; and the numbers in the third column muſt be kept the ſame, the bills always giving theſe right.—After 20, the Table is to be continued unaltered; and the reſult will be, a Table which will give the numbers of the living at all ages in *London* much nearer the truth, but ſtill ſomewhat too high.—Such is the 13th Table in the Appendix.—The ſum of all the numbers in the ſecond column of this Table, diminiſhed by 500, is 20,750. For every 1000 deaths, therefore, in *London*, there are, according to this Table, 20,750 living perſons in it; or for every ſingle death, 20¾ inhabitants. It was before ſhewn, that the number of inhabitants in *London* could not be ſo great as 25 times ¾ the deaths. It now appears, (ſince the numbers in the ſecond column of this Table are too high) that the number of inhabitants in *London* cannot be ſo great as even 20 times ¾ the deaths. And this is a concluſion which, I believe, every one who will beſtow due attention on what has been ſaid, will find himſelf forced to receive. It will not be amiſs, however, to confirm it by the following fact, the knowledge of which I owe to the particular enquiry, and kind information of Mr. *Harris*, the ingenious maſter of the Royal Mathematical School in Chriſt-Church Hoſpital.—The average of lads in this ſchool has, for 30 years paſt, been 831. They are admitted at all ages between ſeven and eleven; and few ſtay beyond 16. They are, therefore, in general lads between the ages of eight and 16. They have better accommodations than it can be ſuppoſed children commonly have; and about 300 of them have the particular advantage of being educated in the country. In ſuch circumſtances it may be well reckoned, that the proportion of children dying annually, muſt be leſs than the general proportion of children dying annually at the ſame ages in *London*. — The fact is, that, for the laſt 30 years, $11\frac{4}{5}$ have died annually; or *one* in $70\frac{2}{5}$.

According to Table XIII, *one* in 73 dies between 10 and 20, and *one* in 70 between eight and 16. That Table, therefore, probably gives the decrements of life in *London*, at theſe ages, too little, and the numbers of the living too great: And, if this is true of theſe ages, it muſt be true of all other ages *under* 20; and it follows demonſtrably, in conformity to what was before ſhewn, that more people ſettle in *London* after 20, than the $\frac{1}{4}$ I have ſuppoſed; and that from 20 to at leaſt 30 or 35, the numbers of the living are given too great, in proportion to the decrements of life.

In this Table the numbers in the ſecond column are doubled at 20, agreeably to what really happens in *London*; and the ſum of the numbers in this column diminiſhed by half the whole number of deaths, gives the *expectation* of life, not of a child juſt born, as in other Tables, but of all the inhabitants of *London* at the time they enter it, whether that be at birth, or at 20 years of age. The *expectations*, therefore, and the *values* of *London* lives under 20, cannot be calculated from this Table. But it may be very eaſily fitted for this purpoſe, by finding the number of births which, according to the given decrements of life, will leave 494 alive at 20; and then adapting the intermediate numbers in ſuch a manner to this radix, as to preſerve all along the number of the living, in the ſame proportion to the numbers of the dead. This is done in the 14th Table in the Appendix; and this Table may, I fancy, be recommended as better adapted to the preſent ſtate of *London* than any other Table. The values of lives, however, deduced from it, are in general nearly the ſame with thoſe deduced by Mr. *Simpſon*, from the *London* bills as they ſtood 40 years ago. The main difference is, that after 52, and in old age, this Table gives them ſomewhat lower than Mr. *Simpſon*'s Table.

It has ſufficiently appeared, what judgment we are to form of the values of lives thus deduced. During the greateſt part of the interval of life, in which the annual recruits that keep up *London* come to it, theſe values err on the ſide of *exceſs*; and after that interval, they err, perhaps, a little on the ſide of *defect* (*a*), on account of retirements from *London* in the laſt ſtages of life.

(*a*) I have not taken into account the effect of migrations *from* towns, on Tables formed in the manner I have explained;

The number of inhabitants in *London* may alſo be learnt from what has been offered, more

explained; becauſe, towns in general being kept up by recruits from the country, the migrations *from* them are of little conſequence, compared with the migrations *to* them.—Thus; in LONDON, it appears, from the much greater number of deaths between 40 and 50, than in any other equal interval of life after 10, that more people come to it than leave it, at every age between 20 and 50. After 50, it is probable, that the contrary happens. But, it ſhould be conſidered, that *emigrants* from LONDON after 50, are chiefly perſons who, having got fortunes in buſineſs, chuſe to leave off, and to ſpend the latter part of their lives in country retirements. But how few are theſe compared with the multitudes who, tho' poſſeſſed of good fortunes, never retire; and with the bulk of the inhabitants in lower ſtations, who never can be able, without the greateſt inconveniencies, to quit the ſettlements by which they are ſupported? It is, however, likely, that retirements from LONDON are now more numerous than they ever were; and that they have *ſome* effect on the bills of mortality, and on *Tables* formed from them; by cauſing theſe Tables to give the number of the living too little, in compariſon with the *decrements* of life, at every age, from that at which the migrations *to* and *from London* become equal, to the age at which the latter ceaſe.—To explain this; let us ſuppoſe, that none ſettle in LONDON after 50; but that, between 35 and 50, as many come to it as retire from it at all ages after 35; and that theſe retirements ceaſe at 70. In this caſe, the Tables will give the proportion of the living to the decrements of life too high *'till* 35. *At* 35, this proportion will be given right. *After* 35, it will begin to be given too low; and this error will increaſe 'till 50; from which age it will decreaſe gradually 'till it vaniſhes at 70: And after 70, the Tables will be exactly right again.—This is the exact ſtate of the effect of retirements from *London*, on the *London* Table of Obſervations. But this effect appears, indeed, to be inconſiderable; for, after more nearly than by any method which has been hitherto taken. It cannot, it has been ſhewn, exceed 20 times ¼ the number of annual deaths. Could, therefore, the annual deaths be aſcertained, we ſhould know the number of inhabitants within pretty narrow limits. But the omiſſions in the bills are ſuch, that it is not poſſible to aſcertain, with exactneſs, the annual deaths. Dr. *Brakenridge* ſuppoſed theſe omiſſions to amount to 2000 annually. The reſult of a very minute enquiry by Mr. *Maitland* is, that in the year 1729, they amounted to 3038. But they are probably now much more conſiderable, than either of theſe writers have reckoned them (*a*). Let them be 6000; and the number of inhabitants will be 601,750 at moſt.

All the preceding Obſervations are, it is plain, applicable to bills of mortality for towns in general; and point out the way of deducing from them genuine Tables of Obſervations,

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after 50, the values of lives by the *London* Table, are continually approaching nearer and nearer to the ſame values by other Tables; which could not happen were retirements attended with any *great* effect.—It is proper to add, that in ſumming up, as above-explained, the numbers of the living, in order to find the number of inhabitants in *London*, the circumſtance that theſe numbers may be too ſmall for ſome years after 40 or 50, in conſequence of retirements, is, undoubtedly, much more than balanced by their being given too high between 20 and 40.

(*a*) Vid. Preface to a Collection of the Bills of Mortality from 1657 to 1758, p. 4, &c.

ſervations, which ſhall give the true probabilities and values of lives, and the true number of inhabitants, in the town whoſe bills are given.—I ſhall beg leave to confirm and illuſtrate this, in the particular caſe of the town of NORTHAMPTON.

In this town, containing four pariſhes, namely, *All-Saints*, *St. Sepulchre's*, *St. Giles*, and *St. Peter's*, an account has been kept ever ſince the year 1741, of the number of males and females that have been chriſtened and buried (Diſſenters included) in the whole town. And in the pariſh of *All-Saints*, containing the greateſt part of the town, an account has been kept ever ſince 1735, of the *ages* at which all have died there.

In 1746, an account was taken of the number of *houſes*, and of *inhabitants* in the town. The number of *houſes* was found to be 1083; and the number of *inhabitants* 5136.—In the pariſhes of *All-Saints* and *St. Giles*, the number of *male* and *female heads* of families, *ſervants*, *lodgers*, and *children*, were particularly diſtinguiſhed.—The *heads of families* were, 707 males; and 846 females.—*Children*, males 624; females 759.—*Servants*, males 203; females 280.—*Lodgers*, males 137; females 287.—In *St. Peter's*, males 99; females 129. —In *St. Sepulchre's*, *adults* 638; *children* 427. In this pariſh the ſexes were not diſtinguiſhed.

The *Chriſtenings* and *Burials* in the *whole town* for 28 years, from 1741 to 1770, have been as follows.

Chriſtened	Males	2361	4649	— Annual medium 155
	Fem.	2288		
Buried	Males	2869	5747	— Annual medium 191
	Fem.	2878		

In the pariſh of *All-Saints,* from 1735 to 1770, or 36 years,

Chriſtened	Males	1632	3242	—Annual medium 90
	Fem.	1610		
Buried	Males	1856	3690	—Annual medium 102½
	Fem.	1834		

Of theſe died,

Under 2 years of age —	1206
Between 2 and 5 —	276
Between 5 and 10 —	155
Between 10 and 20 —	155
Between 20 and 30 —	297
Between 30 and 40 —	257
Between 40 and 50 —	297
Between 50 and 60 —	300
Between 60 and 70 —	293
Between 70 and 80 —	285
Between 80 and 90 —	155
Between 90 and 100 —	14
Total	3690

A Table formed from theſe *data* in the manner of Table XII; or, on the ſuppoſition, that all who *die* in *Northampton* were *born* there, would give the expectation of a child juſt born 28.83 years; or, the proportion of the inhabitants to the annual deaths, as 28.83 to 1. It has been ſhewn, that this proportion, in a place where the burials exceed the births, muſt be *greater* than the *true* proportion of the number of inhabitants to the annual deaths: And this appears to be the real caſe. For the bills ſhew, that, from 1741 to 1750, or for 10 years, about the time when the number of inhabitants was 5136, the annual medium of burials was 197.5; which, multiplied by 28.83, gives 5693; or a 9th part more than the *true* number.

A Table formed in the manner of Table XIII, would give the proportion of inhabitants to the annual deaths, as 26.41 to 1; and this makes the inhabitants 5216; or very near the true number.

The IVth Table, in the *Appendix*, is formed in the ſame manner with Table XIV, for *London*: And this is the genuine Table of Obſervations for *Northampton*, from which may be calculated the true probabilities and values of lives, at all ages, in that town.

At Norwich, bills of mortality, of the ſame kind with thoſe in *London* and *Northampton*, have been kept for many years. I have been favoured with a copy of theſe bills for 30 years, from 1740 to 1769. The annual medium of *chriſtenings*, during this period, has been 1057 (*a*), of *burials* 1206. And from hence, together with the account of the numbers dying in the ſeveral decads of life, after 10, I have formed Table V, which ſhews the true probabilities of life in this town.

The following particulars ſeem to deſerve notice here.

Firſt. Had theſe Tables been formed from the NORTHAMPTON and NORWICH bills, for no longer time than any 10 years taken together, of the periods I have mentioned; they would have given the probabilities and values of lives nearly the ſame. Theſe Tables, therefore, are founded on a ſufficient number of Obſervations; and it appears, that there is an invariable law which

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(*a*) In this regiſter all that die before baptiſm, and alſo all that are born and die among *Quakers*, *Jews*, &c. are omitted. There are alſo ſome other omiſſions; and the true annual medium of births and burials muſt be greater than they are given in the bills. But this will have no effect on a Table of Obſervations, ſuppoſing the *proportions* of the births to the burials, and of the numbers dying in the different ſtages of life, given right. —It is proper I ſhould mention further here, that theſe bills give only the whole number of children dying under 10, without ſpecifying the numbers dying under two years of age, between 2 and 5, and between 5 and 10, as in other bills. I have, therefore, in forming the Table for NORWICH, ſuppoſed the proportions of theſe numbers the ſame that they are at NORTHAMPTON.

governs the waſte of human life in theſe towns.—The ſame remark might be made concerning LONDON (*a*). See p. 251.

Secondly. An account was taken at SHREWSBURY, in 1750, of the *whole* number of inhabitants; diſtinguiſhing, particularly, the number at the age of 21 and upwards.—The former number was 8141; and the latter, 5187.—According to a Table formed for NORTHAMPTON, in the ſame manner with Table XIII, for LONDON, the whole number of the living is to the number of the living at 21 and upwards, as 26,411 to 16,586; that is, as 8141 to 5113.—According to a like Table for NORWICH, theſe numbers are to one another, as 24,500 to

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(*a*) Some have entertained a very wrong notion of the imperfections in the LONDON bills. They do, indeed, give the *whole* number of births and deaths much too little; but the concluſions with reſpect to the probabilities of life in LONDON, and the proportion of inhabitants dying annually, depend only (agreeably to the obſervation in the laſt note) on the *proportions* of the numbers dying in the ſeveral diviſions of life; and theſe are given *right* in the LONDON bills.—For firſt. There ſeems nothing in this caſe, that can be likely to cauſe the deficiencies in the bills to fall in one diviſion of life more than in another: But what decides this point is, that theſe proportions, as given by the bills for *any* ten, or even *any* five years, come out nearly the ſame with one another; and always very different from the proportions given by other bills.—There are no other variations, than ſuch as muſt ariſe from the fluctuations of LONDON, as to increaſe and decreaſe; and alſo from ſome improvements in its ſtate, which have lately taken place. See Eſſay I. p. 190, 191, 201.

15,680; that is, as 8141 to 5210.—Theſe Tables, therefore, give the proportion of the *whole* number of inhabitants, to the number of the living at 21 and upwards, almoſt exactly the ſame with the true proportion, as it is at SHREWSBURY (*a*): And this affords a kind of demonſtration of the rectitude of the principles on which theſe Tables have been formed.

In the pariſh of HOLY-CROSS near SHREWSBURY, an account was taken, in 1760 and 1770, of the *whole* number of inhabitants; diſtinguiſhing, *both* times, the number at the age of 70 and upwards; and the *laſt* time, the number at 10 and upwards: And, I find, that a Table formed from the *Regiſter* of this pariſh, mentioned p. 192, gives, likewiſe, *theſe* numbers as nearly the ſame as could poſſibly be expected.

But further.—The number of inhabitants, not reckoning children, in the pariſhes of *St.*

(*a*) The annual medium of births at SHREWSBURY, for 7 years, from 1762 to 1768, was 301; of burials, 329. It appears, therefore, that one in 24¾ of the inhabitants die annually. But it ſhould be remembered, that in 1766, the ſmall-pox and meaſles increaſed very much the mortality in this town; and I find alſo, that, ſince 1750, a nurſery for *foundlings* from LONDON, was eſtabliſhed here; and that in 1768 this nurſery contained 660 children and ſervants. It ſeems, therefore, probable, that the true *medium* of burials about the year 1750, muſt have been leſs than 329; and that the proportion of inhabitants dying annually, may not be much greater than it is at NORTHAMPTON; or 1 in 26.41.

Giles and *All-Saints*, NORTHAMPTON, was, in 1746, 2460; and the *whole* number of inhabitants in theſe two *pariſhes* was 3843. See p. 254.—In the account I have received, the particular age at which the limit of childhood was fixed in taking this ſurvey, is not mentioned; but there is ſufficient reaſon to believe, that it was 21: And, taking this for granted, the number of inhabitants, not children, will come out, (by ſuch a Table for NORTHAMPTON, as Table XIII for LONDON) 2414; or, nearly the ſame with the number *really* found in theſe pariſhes.—Had this number been computed, from a Table formed for NORTHAMPTON, in the manner of Table XII, *Appendix*, it would have come out only 2176. This remark is applicable to the Table for *Breſlaw*, formed by Dr. *Halley*, compared with the ſame Table, corrected for all the ages under 20 (*a*), by the rule, p. 246.

(*a*) I have given Dr. *Halley*'s Table in the Appendix juſt as he framed it. A correction of it might be made from the proportion of births to burials, mentioned p. 239. And it would then appear, that a 25th part of the inhabitants at *Breſlaw* die annually; and that half the number born die there under ſix, as well as at *Norwich*. This Table, as we now have it, makes half live to 16; but the account mentioned in the note, page 240, ſhews this not to be the truth. It likewiſe makes the number of inhabitants at SHREWSBURY, above the age of 21, to be 4730; and in the pariſhes of *All-Saints* and *St. Giles* in NORTHAMPTON, 2230. It gives, therefore, theſe numbers wrong; whereas, as obſerved above, a corrected Table would give them true.

The neceſſity, therefore, of this correction is verified by facts; and it appears, abundantly, that the Tables I have given for NORTHAMPTON and NORWICH may be depended on.

But, thirdly. In comparing theſe two Tables, it may be obſerved, that there is a difference between them in favour of NORTHAMPTON, *fewer* dying there in childhood, and *more* in old age. The ſame would be found to be true, were the NORTHAMPTON Table to be compared with a corrected BRESLAW Table. It appears, therefore, agreeably to what might have been expected, that NORTHAMPTON, being a ſmall town compared with BRESLAW and NORWICH, is leſs unfavourable to health and longevity. The difference, however, is not conſiderable. After the age of 20, there is a ſtriking conformity between all the three Tables, which gives them great weight and authority.

Further. It ought to be particularly noted, that theſe Tables prove, the *decrements* of life in moderate towns, to be nearly *equal* thro' moſt of its ſtages. At NORTHAMPTON it appears that, of a given number of perſons alive at 20, the ſame number die every year 'till 78, without any interruption worth notice, except between the ages of 30 and 40.—A like uniform decreaſe in the probabilities of life appear in the BRESLAW and NORWICH Tables; but not ſo remarkably. It was this circumſtance in the BRESLAW Table, that led Mr. *De Moivre* to the *Hypotheſis*, deſcribed in p. 2, and ſo often mentioned in this work.—The values of lives, I have ſaid, deduced from this *Hypotheſis*, agree ſo nearly with the ſame values deduced immediately from the Tables, that it is ſcarcely worth while to diſtinguiſh them. But that every one may be able to judge of this for himſelf, I have calculated (*a*) the following Table.

	Value of a life at the age	By *Breſlaw* Table.	By *Norwich* Table.	By *Northampton* Table.	By Mr. *De Moivre's Hypotheſis.*
	12	17.617	17.48	17.20	16.69
Reckoning intereſt at 4 *per cent.*	20	16.49	16.41	15.93	15.89
	30	14.77	15.15	14.85	14.68
	40	12.90	13.36	13.10	13.19
	50	10.87	11.13	11.25	11.34
	60	8.58	8.54	9.02	9.01
	70	5.59	5.99	6.26	6.06
	75	4.21	4.86	4.79	4.29

It may be obſerved in this Table, that the values, by the *Hypotheſis*, come nearer to the true values by the NORTHAMPTON and NORWICH Tables, than by the BRESLAW Table; and alſo, that, before the age of 60, they are *all* much higher than the values for

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(*a*) Every calculation of this kind may be made without much labour, by a rule explained in note (O) *Appendix*.

the ſame ages in LONDON by Table X; the inhabitants of *London*, (as Mr. *De Moivre* obſerves) being "for cauſes (*a*) too well known, "more ſhort-lived than the reſt of mankind." —The *Hypotheſis*, therefore, is by no means applicable to LONDON lives. It is proper to add, that neither can it be applied to the valuation of COUNTRY lives.—It appears, from the regiſter of the pariſh of *Holy-Croſs* (*b*), that the *expectations* of lives there are much greater than the *expectations* by the *Hypotheſis*. —The *expectation* there of a life (*c*)

At 20	is 38	By *Hypoth.* 33	In *Lond.* 28.9
27	33.9	29.5	25.1
30	32	28	23.6
40	25.7	23	19.6
50	20	18	16
60	14.5	13	12.4
70	10	8	8.8

From

(*a*) Doctrine of Chances, p. 347.

(*b*) See Eſſay I. p. 191. —I have not given the Table of Obſervations from whence theſe concluſions are deduced, becauſe it is poſſible, ſome may think 20 years not a period long enough, to afford *data* in this caſe of ſufficient authority. I have in p. 257, mentioned a fact which ſeems to prove the contrary. It is, however, certain, that the ſame regiſter continued 10 or 20 years longer, will afford *data* more to be depended on.

(*c*) The expectation of a child juſt born in this pariſh, is 33. At NORTHAMPTON, $25\frac{1}{2}$. At NORWICH, $23\frac{3}{4}$. In LONDON, 18.—In this pariſh, 1 in 11 dies at 80, and upwards. In NORTHAMPTON; 1 in 22. In NORWICH; 1 in 27. In LONDON; 1 in 40. See Eſſay I. p. 200.

From this compariſon it appears, that the *Hypotheſis*, from 20 to 60, gives nearly the medium

I will add, that the probabilities of life here, appear to be much the ſame, with the probabilities of life among the miniſters and profeſſors in SCOTLAND.—This is a fact of ſome conſequence; and, therefore, I ſhall beg leave to give a brief account of it.

The mean age at which the miniſters and profeſſors enter into benefices and profeſſorſhips in *Scotland*, is reckoned to be 27. Their number is 974. The eſtabliſhment among them for providing for their widows, begun on the 25th of *March* 1744; from which time to *November* 22, 1768, 721 have died: That is, 29.23 annually; or 1 in 33⅓. The *expectation*, therefore, of a life among them, at the age of 27, is 33⅓; which is nearly the ſame with the *expectation*, as given above, of a life of the ſame age in the pariſh of *Holy-Croſs*; and 3½ years more, than the *expectation* of the ſame age by Tables III, IV and V.—Now, the *expectation* at a given age, being compoſed of all the probabilities of life from that age to the extremity of life; there ariſes from hence reaſon for concluding, that the probabilities of life among the miniſters in *Scotland*, cannot differ much in any part of life, from thoſe in this pariſh.—But there is another fact that confirms this obſervation.

The annual average of weddings among the miniſters and profeſſors in SCOTLAND, for the laſt 24 years, has been at moſt 32. The average of married perſons among them, for 17 years ending in 1767, had been 667. This number, divided by 32, gives 20.84, the *expectation* of marriage among them; which is 2¼ years more than the *expectation* of marriage would be, by Dr. *Halley*'s Table, on the ſuppoſition, that all 1ſt, 2d and 3d marriages may be juſtly conſidered as commencing, one with another, ſo early as the age of 30.—The *expectation* of two equal *joint* lives is to the *expectation* of a *ſingle* life of the ſame age, as 2 to 3, by note (L) *Appendix*. It follows, therefore, that among the miniſters in *Scotland*, the *expectation* of a *ſingle* life medium between the *expectations* of LONDON and COUNTRY lives; and for this reaſon it is excellently adapted to general uſe.—After 60, the *expectations* and *values* of lives in LONDON approach nearer and nearer to the *expectations* and *values* of lives in *Northampton*, *Norwich* and *Breſlaw*; 'till, at 70, they come to be almoſt the ſame. This is a circumſtance which, I believe, has not been attended to: And it is the more ſurprizing, as there is no cauſe known, which can produce any error in the values of lives after 60, deduced from the LONDON Table, except migrations from *London*; and the effect of theſe muſt be to *diminiſh* theſe values.

The following obſervations will, perhaps, account for this.

It has been proved, that at leaſt *half* the inhabitants of LONDON, turned of 20 years of age, are *emigrants* to LONDON from the country. So great a change as that, from the *country* air and modes of life, to the air and modes of life in *London*, muſt be particularly

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life at 30 cannot be leſs than 31.26. Moſt probably it is more; on account of the later commencement of marriage in the ſituation of the *Scotch* miniſters. — I reckon alſo, that 27 muſt be leſs than the mean age at which they enter their benefices and profeſſorſhips; meaning by it, not the age on each ſide of which equal numbers enter; but the age at which, the exceſs of the interval of time taken to enter on one ſide, is juſt ſuch as to compenſate the greater numbers who enter on the other ſide. See the concluſion of note (F) *Appendix*.

cularly hurtful to theſe perſons; and, therefore, (except *infants*) it is in them, probably, that the pernicious influence of *London* on its inhabitants chiefly appears. They come in at every age 'till near 50; and this is the reaſon why the deaths continually increaſe in *London* 'till that age; but, after that age, the inhabitants conſiſting chiefly of perſons, who (like men *uſed* to drink) have been *ſeaſoned* to *London*, or with whom it does not happen particularly to diſagree; the number of deaths becomes leſs, and the values of lives begin to approach nearer to the common ſtandard in other towns.

There is one more fact which I ſhall here take notice of; and which deſerves more attention than has been hitherto beſtowed upon it. I mean; "the difference between the "probabilities of life among *males* and *females*, in favour of the latter."

From the account in p. 254, it appears, that at NORTHAMPTON, tho' more *males* are born than *females*, and nearly the ſame number die; yet the number of living *females* is greater than the number of *males*, in the proportion of 2301 to 1770, or 39 to 30. This cannot be accounted for, without ſuppoſing, that *males* are more ſhort-lived than *females*.—One obvious reaſon of this fact is, that *males* are more ſubject to untimely deaths by accidents of various kinds; and alſo, in general, more addicted to the exceſſes and irregularities

which

which ſhorten life. But this is by no means the *only* reaſon. For it ſhould be obſerved, that at NORTHAMPTON the number of *female* children was, in 1746, greater than the number of male children, in the proportion of 759 to 624.—The greater mortality of males, therefore, takes place among *children.*—But this, together with the greater mortality in general of males at all ages, will more particularly appear from the following recital of facts.

In the pariſh of *Holy-Croſs*, *Salop*, the ingenious Vicar, Mr. *Gorſuch*, in 1760, and again in 1770, took the number of male and female inhabitants turned of 70. In 1760, the number of females turned of this age, was 35; of males, 8. In 1770, theſe numbers were, females, 35; males, 26. And for the laſt 10 years 11, out of 365 have died between the ages of 85 and 102; and they were all females.

At BERLIN, it appeared, from the accurate account which was taken of the inhabitants in 1747, and which has been mentioned in p. 219, that the number of *female* citizens exceeded the number of *male* citizens, in the proportion of 459 to 391: And yet, out of this ſmaller number of males, more had died, for 20 years preceding 1751, in the proportion of 19 to 17 (*a*).

(*a*) Vid. *Suſmilch*, p. 8, and p. 32, &c. where a minute account is given of the number of males and females at BERLIN in 1747; and alſo, of the numbers of each ſex that had died from 1722 to 1750.

At EDINBURGH, in 1743, the number of *females* was to the number of *males*, as 4 to 3; (See Eſſay I. p. 211) but the females that died annually, from 1749 to 1758, were to the males, in no higher proportion than $3\frac{1}{5}$ to 3. Before 1749, the bills give the totals of burials, without diſtinguiſhing them into the totals of males and females dying every year.

Mr. *Kerſeboom*, in his Eſſay on the numbers of people in HOLLAND, informs us, that from the Tables of aſſignable Annuities for lives in HOLLAND, which had been kept there for 125 years, wherein the ages of the perſons dying are truly entered; it appears, that females have, in all accidents of age, lived about 3 or 4 years longer than the ſame number of males. See *Philoſophical Tranſactions* abridged, Vol. IX, p. 326.

In Volume the 7th of the *Philoſophical Tranſactions* abridged, Part IV, p. 46, &c. there is an account of the numbers of *male* and *female* ſtill-born children and chryſoms, and of boys and girls under 10, of married men and married women, and of widows and widowers, who died for a courſe of years at *Vienna*, *Breſlaw*, *Dreſden*, *Leipſic*, *Ratiſbon*, and ſome other towns in GERMANY.

He that will take the pains to examine theſe accounts will find that, though in theſe towns the proportion of males and females born is no higher than 19 to 18, yet the proportion of boys and girls (*a*) that die is 8 to 7; and that, in particular, the *ſtill-born* and *chryſom males*, are to the ſtill-born and chryſom *females*, as 3 to 2.

In theſe accounts it appears alſo, that of 7270 *married* perſons who had died in theſe towns (*b*), 4336 were married *men*, and but 2934 married *women*; that is, *three* married *men* died to *two* married *women*.—The ſcheme for making proviſion for the widows and orphans of the miniſters in SCOTLAND, has obliged them to keep an account of the number of *weddings* among them, and the number of *widows* left annually; and it appears, from the *reports* of the *truſtees* for carrying this ſcheme into execution, that the annual medium of *weddings* (*c*), is (as obſerved in the note, p. 264) at moſt 32. And the annual medium

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(*a*) In the accounts from *Breſlaw* it is particularly mentioned, that by *boys* and *girls* are meant children to 10 years of age, of whom, for 8 years from 1717 to 1725, *ſeven* males died to *ſix* females, excluſively of the *ſtill-born* and *chryſoms*.

(*b*) In *Breſlaw* alone, for the eight years mentioned in the laſt note, 1891 married men died, to 1196 married women; that is 5 to 3.—In *Dreſden* alone, for five years, theſe numbers were 1080 and 849.

(*c*) The annual medium of weddings, among the miniſters admitted to benefices, has been, for 24 years from the commencement of the ſcheme, 27. Beſides theſe I find there have been 4 weddings annually among them, *before* admiſſion to benefices. The whole annual medium, therefore, is no more than 31. But I have ſuppoſed it 32, in order to go upon more ſure grounds.

medium of widows, who have come upon the ſcheme for 24 years, is 20. Of 32 marriages then contracted annually, 20 become extinct by the deaths of *huſbands*; and but 12 by the deaths of *wives*. That is; among the miniſters and profeſſors in SCOTLAND, 20 married men die to 12 married women; or 5 to 3. It appears, therefore, that there is the chance of 3 to 2, and in ſome circumſtances even a *greater* chance, that the *woman* ſhall be the ſurvivor of a marriage, and not the *man*. In order to account for this by the difference of age between men and their wives, this difference ought to be at leaſt 12 years (*a*). That is; ſuppoſing the mean age at which women marry to be 23, the mean age at which men marry ought to be 35. But this ſeems to exceed the bounds of credibility; and, therefore, very probably, the greater mortality of males muſt operate in this caſe.

It is further obſervable in the accounts from *Germany*, to which I have referred, that the number of *widows* dying annually, is four times the number of *widowers* (*b*); and, as *widows*

(*a*) The chance of ſurvivorſhip between two perſons aged 21 and 34, is nearly 3 to 2 in favour of the former. There is the ſame chance of ſurvivorſhip between 25 and 37; and 28 and 39. This may be learnt from Problem XVI, in Mr. *De Moivre*'s Treatiſe on *Life-Annuities*.

(*b*) In *Dreſden* alone, the number of *widows* who died, in four years, was 584. The number of *widowers*, 149. That is; 4 to 1.—It appears from note (*b*) in the laſt page, *widows* are certainly, one with another, ſeveral years younger than *widowers*; it may be concluded from hence, that the number of the former in life together could not be leſs than five times the latter.—This fact is likewiſe confirmed, by the obſervations which have been made among the miniſters in *Scotland*. At the commencement of the ſcheme which I have ſo often had occaſion to mention, an account was taken of the number of the *widows* of miniſters in the whole country. 364 were counted; and, probably, the true number was greater. See p. 95 and 96. On the contrary; the number of *widowers* among the miniſters has, one year with another, been ſcarcely 90; that is, not ſo much as a *quarter* of the number of *widows*.—It may be eaſily ſeen, and it would not be difficult to demonſtrate, that neither the greater number of perſons left widows, nor any probable ſuppoſition concerning the greater frequency of marriages among widowers, can completely account for this, without admitting the *greater mortality of males*.—This, therefore, appears, on the whole to be a fact

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page, that the chance of ſurvivorſhip in this city in favour of the *wife*, is leſs than among the miniſters in *Scotland*.—Does not this fact afford a reaſon, additional to that mentioned above, and in p. 92, 93, 94, &c. and notes (A) and (F), for *believing*, or at leaſt *ſuſpecting* that the number of *widows* on the *Scotch* eſtabliſhment, if marriage does not decline among the miniſters, will not at laſt be found to be ſo little as 400?

well eſtabliſhed: And it follows from it, that in order to calculate the values of *Life-Annuities* and *Reverſions* with exactneſs, there ought to be diſtinct Tables of the probabilities of life for *males* and *females*. But there are no ſuch Tables extant; nor, indeed, has it been ſuſpected, that there is ſo much occaſion for them as the facts I have mentioned ſeem to ſhew. All that is neceſſary to obtain the proper *data* for forming ſuch Tables is, that the *ſexes* as well as the *ages* of the dead ſhould be ſpecified in the bills; and this is an improvement of bills (*a*) of mortality which would give little trouble, and which, therefore, I hope, will be ſome time or other made.

It has been obſerved, that the author of nature has provided, that more *males* ſhould be born than *females*, on account of the particular waſte of *males*, occaſioned by wars and other cauſes. Perhaps it might have been added, that this proviſion had alſo in view, that particular weakneſs or delicacy in the conſtitution of males, which makes them more ſubject to mortality; and which, con-

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(*a*) This improvement would be rendered more complete, by diſtinguiſhing the *males* that die, under the denominations of *married men*, *widowers*, and *batchelors*; and the *females*, under the denominations of *married women*, *widows*, and *virgins*.—The uſe I have made of ſome accounts of this kind which have been kept in *Germany*, ſhews that this would be of conſiderable ſervice.

ſequently, renders it neceſſary, that more of them ſhould be produced, in order to preſerve in the world a due proportion between the two ſexes.

In the courſe of this Eſſay, it has often appeared, that I have been particularly indebted to an information which I have received from NORTHAMPTON.—I ſhould be inexcuſable, did I not mention, that I owe this information to Mr. *Lawton*, an ingenious gentleman in that town, who has preſerved the bills of mortality there with much care, and been very obliging in communicating them to me.—It is much to be deſired, that like accounts were kept in every town and pariſh. It would be extremely agreeable to learn from them the different rates of human mortality in different places, and the number of people and progreſs of population in the kingdom. The trouble of keeping them would be trifling; but the inſtruction derived from them (*a*), would be very important.—I have already propoſed one improvement of ſuch accounts. I will add, that they would be ſtill more uſeful, did they give the ages of the dead after 10, within periods of *five*, inſtead of *ten* years.—During every period, ſo ſhort as *five* years, the decrements of life may, in conſtructing Tables, be ſafely

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(*a*) See Eſſay I. p. 207, 208.

taken to be *uniform*. But this cannot be equally depended on, in periods ſo long as ten years.

There is yet another improvement of theſe accounts, which I will take this opportunity to mention. They ſhould contain not only a liſt of the diſtempers of which all die, like that in the *London* bills; but they ſhould ſpecify particularly the numbers dying of theſe diſtempers, in the ſeveral diviſions of life. Accurate regiſters of mortality kept in this manner, in all parts of the kingdom; and compared with records of the ſeaſons, and of the weather, and with the particular circumſtances which diſcriminate different ſituations, might contribute, more than can be eaſily imagined, to the increaſe of *phyſical* knowledge. — But to proceed no farther in theſe Obſervations; I ſhall now beg leave to ſhut up this whole work with the following general reflexion.

I have repreſented particularly, the great difference between the probabilities of human life in towns and in country pariſhes; and from the facts I have recited, and the obſervations I have made, it appears, that the further we go from the artificial and irregular modes of living in great towns, the fewer of mankind die in the *firſt* ſtages of life, and the more in its *laſt* ſtages. The lower animals (except ſuch (*a*) as have been taken under human management) ſeem in general to enjoy the full period of exiſtence allotted them, and to die chiefly of old age: And were any obſervations to be made among *ſavages*, perhaps the ſame would be found to be true of them.—DEATH is an evil to which the order of providence has ſubjected every inhabitant of this earth; but to man it has been rendered unſpeakably more an evil than it was deſigned to be. The greateſt part of that black catalogue of diſeaſes which ravage human life, is the off-ſpring of the tenderneſs, the luxury, and the corruptions introduced by the vices and falſe refinements of civil ſociety (*b*). That delicacy which is injured

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(*a*) Calves are the only animals taken under our peculiar care immediately after birth; and, in conſequence of then adminiſtring to them the ſame ſort of phyſic that is given to *infants*, and treating them in other reſpects in the ſame manner, it is probable, that more of them die ſoon after being born, than of *all* the other ſpecies of animals, which we ſee in the ſame circumſtances. See the *Comparative View of the State and Faculties of Man with thoſe of the Animal World*, p. 23.—It is, indeed, melancholy to think of the havock made among the human ſpecies by the unnatural *cuſtoms* as well as the *vices*, which prevail in poliſhed ſocieties. I have no doubt, but that the cuſtom, in particular, of committing infants, as ſoon as born, to the care of *foſter-mothers*, deſtroys more lives than the ſword, famine and peſtilence put together.

(*b*) The ingenious and excellent writer quoted in the laſt note, obſerves, that the whole claſs of diſeaſes which ariſe from catching cold, are found only among the civilized part of mankind, p. 51.—And, concerning that loſs jured by every breath of air, and that rottenneſs of conſtitution which is the effect of intemperance and debauchery, were never intended by the author of nature; and it is impoſſible, that they ſhould not lay the foundation of numberleſs ſufferings, and terminate in premature and miſerable deaths. — Let us then value more the ſimplicity and innocence of a life agreeable to nature; and learn to conſider nothing as ſavageneſs but malevolence, ignorance and wickedneſs. The order of nature is wiſe and kind. In a conformity to it conſiſts health and long life; grace, honour, virtue and joy. But nature turned out of its way will always puniſh. *The wicked ſhall not live out half their days.* Criminal exceſſes embitter and cut ſhort our *preſent exiſtence*; and the higheſt authority has taught us to expect, that they will not only kill the *body*, but the *ſoul*; and deprive of an EVERLASTING EXISTENCE.

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loſs of all our higher powers which often attends the decline of life, and which is ſo humiliating to human pride; he obſerves, that it exhibits a ſcene ſingular in nature, and that there is the greateſt reaſon to believe, that it proceeds from adventitious cauſes, and would not take place among us if we led natural lives, p. 62.

APPENDIX.

Note (A). See Queſtion III. Page 11.

LET E be any given expectation of life; and $\frac{4E-x}{4E}\times px$ will be the number of perſons alive at the end of x years, ariſing from p perſons left annually as widows, (or added annually to a town or ſociety) at the age whoſe *expectation* is E. The *maximum*, therefore, is always pE—. In Mr. *De Moivre's Hypotheſis*, E is always $\frac{1}{2}$ the difference between the given age and 86. See the note page 2, and the latter end of the note in page 37. See likewiſe the beginning of the Firſt Eſſay, and note (L) in this Appendix, where the inveſtigation of this rule will be given.

It will not be amiſs to give the following example of the application of this rule.

At the time of the commencement of the ſcheme, among the miniſters and profeſſors in SCOTLAND, for making proviſion for their widows, it was neceſſary, that a calculation ſhould be made of the number of widows that would be upon the ſcheme at the end of every year, till they came to a *maximum*, on the ſuppoſition that, (agreeably to what particular enquiry had ſhewn to have happened for many preceding years,) 20 new widows would be left every year. In order to make this calculation, let 4 of the 20 widows be ſuppoſed to be under 32 years of age when left; and let 28 be ſuppoſed their mean age. Let the ſame number be left between 32 and 39, and let 35 be their mean age; between 39 and 47, and 43 their mean age; between 47 and 57, and 52 their mean age; between 57 and the extremity of life, and 63 their mean age. The number in life together, to which, in 10 years, 4 widows left annually at the age of 28 will grow, is, by the rule, (E being 29) $\frac{116-10}{116} \times 40$, or 36.55——The number alive at the end of 20 years, will be $\frac{116-20}{116} \times 80$, or 66.2. At the end of 30 years, the number alive will be 89; of 40 years, 104.82; of 58 years 116——Theſe numbers, found in the ſame way, for the 2d claſs, (E being 25.5), at the end of 10, 20, 30, 40, and 51 years, will be 36.7—64.31—84.7—97.25—102——for the 3d claſs, (E being 21.5) at the end of 10, 20, 30, 40, and 43 years, 35.34—61.4—78.13—85.6—86——For the 4th claſs, (E being 17) at the end of 10, 20, 30, and 34 years, 34.11—56.47—67—68——For the 5th claſs, (E being 11.5) at the end of 10, 20, and 23 years, 31.3—45.2—46——The whole number, therefore, conſiſting of all the claſſes, will come to a *maximum* nearly in 58 years; and the totals in life, at the end of 10, 20, 30, 40, 50, and 58 years, will be 173.37—293.58—364.83—401.67—418.

Theſe determinations ſuppoſe none to marry. In 10 years, from 1757 to 1767, I have been informed, that but 9 widows married. Let us then ſuppoſe, that one widow of the firſt claſs marries every year; and let all that marry, be ſuppoſed to continue, one with another, 5 years in widowhood before they marry. On theſe ſuppoſitions, the foregoing totals will, at the end of the ſame periods of years, be 169.23—282—347.5—380.47—394.

Theſe calculations are made from Mr. *De Moivre*'s Hypotheſis. Had they been made exactly from Dr. *Halley*'s table, or any other of the Tables I have given at the end of this work, except the *London* one, the reſults would have been very nearly the ſame.—It appears, that the probabilities of life, among theſe widows, are greater than thoſe given by theſe tables. See the Laſt Eſſay, pages 263, 264 266, 270, &c. The effect of this muſt be, to raiſe the *maximum* without ſenſibly increaſing the numbers in life for the firſt 20 or 25 years: and its effect *may* be, to raiſe the *maximum*, and at the ſame time even to diminiſh theſe numbers.

Twenty-five years have now elapſed ſince the commencement of this ſcheme; and the number of widows living every year have, in fact, correſponded to the laſt numbers I have given, as nearly as could poſſibly be expected in an affair of this nature.

Note (B). Queſtion VI. Page 21.

LET r ſignify the ſum of 1*l.* and its intereſt, for one year. The value of a life, whoſe complement is n, being (by Mr. *De Moivre* on *Annuities*, 4th edition, page 14. and p. 100.) $\frac{n-1}{nr}+\frac{n-2}{nr^2}+\frac{n-3}{nr^3}+\frac{n-4}{nr^4}$, &c. the preſent value of the remainder of it after *two* years muſt be $\frac{n-3}{nr^3}+\frac{n-4}{nr^4}$, &c. which is equal to $\frac{1}{r^2}\times\frac{n-2}{n}\times$

$$\overline{\frac{n-3}{\overline{n-2}\,r}+\frac{n-4}{\overline{n-2}\,r^2}+\frac{n-5}{\overline{n-2}\,r^3}}, \text{ \&c.}$$

Now $\frac{1}{r^2}$ is the preſent value of 1*l.* due at the end of two years. $\frac{n-2}{n}$ is the probability that a life, whoſe complement is n, ſhall continue two years, and $\frac{n-3}{\overline{n-2}\,r}+\frac{n-4}{\overline{n-2}\,r^2}+\frac{n-5}{\overline{n-2}\,r^3}$, &c. is the value of a life two years older than the life whoſe complement is n. And, therefore, (ſince any number of years leſs than n may be ſubſtituted for two years) the firſt rule given in this queſtion is right.

The ſame proceſs, applied to joint lives, will demonſtrate what is ſaid in the *Scholium.*

Note (C). See Queſtion VII. Page 22.

LET the complements of any two aſſigned lives be *n* and *m*. The preſent value of the firſt poſſible payment of an annuity to be enjoyed by the life whoſe complement is *n*, provided *both* lives continue 7 years, and the life, whoſe complement is *n*, ſurvives the other *after* that term, is the probability, that the life of the expectant ſhall continue 8 years, and the other life 7 years and then fail in the 8th year, multiplied by $\frac{1}{r^8}$, or by 1*l.* diſcounted for 8 years.—The probability that the life of the *expectant* ſhall continue 8 years is $\frac{n-8}{n}$. The probability that the *other* life ſhall continue 7 years is $\frac{m-7}{m}$. The probability that it ſhall continue 7 years, and fail in the 8th year, is $\frac{m-7}{m} \times \overline{1 - \frac{m-8}{m-7}} = \frac{1}{m}$. The probability, therefore, that the life of the *expectant* ſhall continue 8 years, and the other life 7 and fail in the 8th year, is $\frac{n-8}{n} \times \frac{1}{m}$; and the preſent value of the firſt poſſible payment of the annuity ſuppoſed, is $\frac{n-8}{nr^8} \times \frac{1}{m}$. See *The Doctrine of Annuities* by Mr. *Simpſon*, p. 6—15, or his *Select Exerciſes*, p. 315, &c.——In like manner, the preſent value of the 2d payment, at the end of the 9th year, may be found to be $\frac{n-9}{nr^9} \times \frac{m-7}{m} \times \overline{1 - \frac{m-9}{m-7}}$, or $\frac{n-9}{nr^9} \times \frac{2}{m}$. and the preſent value of all the poſſible payments, $\frac{1}{r^7} \times \overline{\frac{n-8}{nr} \times \frac{1}{m} + \frac{n-9}{nr^2} \times \frac{2}{m} + \frac{n-10}{nr^3} \times \frac{3}{m}}$, &c. But this ſeries is equal to $\frac{1}{r^7} \times \frac{n-7}{n} \times \frac{m-7}{m} \times \overline{\frac{n-8}{\overline{n-7}\,r} \times \frac{1}{m-7} \times \frac{n-9}{\overline{n-7}\,r^2} \times \frac{2}{m-7} + \frac{n-10}{\overline{n-7}\,r^3} \times \frac{3}{m-7}}$, &c. Now $\frac{n-8}{\overline{n-7}\,r} \times \frac{1}{m-7} + \frac{n-9}{\overline{n-7}\,r^2} \times \frac{2}{m-7}$, &c. is the value of an annuity for a life ſeven years older than the expectant, after another life ſeven years older than the life whoſe complement is m. $\frac{n-7}{n} \times \frac{m-7}{m}$ is the probability that both the aſſigned lives ſhall continue 7 years. And $\frac{1}{r^7}$ is the value of 1*l.* due at the end of 7 years. The rule, therefore, given for ſolving this queſtion, is right.

This demonſtration, as well as that in the laſt note, is, for the ſake of more eaſe and clearneſs, applied to the hypotheſis of an equal decrement of life. It does not, however, depend upon it, but may be applied to any table of obſervations.

Note (D). Queſtion IX. Page 29.

LET the complement of any two aſſigned lives be n and m, and the given term be *ſeven* years, as in note (C). The probability that the former life (ſuppoſed to be the life in expectation) ſhall laſt 8 years, is, by Mr. *De Moivre*'s Hypotheſis, $\frac{n-8}{n}$; and the probability that the latter life ſhall fail in 8 years, is $\frac{8}{m}$; and the firſt payment of the annuity mentioned in this queſtion, depends on the happening of *both* theſe events, the probability of which is $\frac{n-8}{n} \times \frac{8}{m}$.

The preſent value, therefore, of the firſt poſſible payment of the annuity is $\frac{n-8}{nr^8} \times \frac{8}{m}$.——In like manner; the preſent value of the *ſecond* poſſible payment is $\frac{n-9}{nr^9} \times \frac{9}{m}$; and of all the payments, $\frac{n-8}{nr^8} \times \frac{8}{m} + \frac{n-9}{nr^9} \times \frac{9}{m} + \frac{n-10}{nr^{10}} \times \frac{10}{m}$, &c. But $\frac{n-8}{nr^8} \times \frac{8}{m} = \frac{n-8}{nr^8} \times \frac{1}{m} + \frac{n-8}{nr^8} \times \frac{7}{m}$; and $\frac{n-9}{nr^9} \times \frac{9}{m} = \frac{n-9}{nr^9} \times \frac{2}{m} + \frac{n-9}{nr^9} \times \frac{7}{m}$. The foregoing ſeries, therefore, is equal to the two ſeries's $\frac{1}{r^7} \times \overline{\frac{n-8}{nr} \times \frac{1}{m} + \frac{n-9}{nr^2} \times \frac{2}{m} + \frac{n-10}{nr^3} \times \frac{3}{m}}$, &c. and

$\frac{1}{r^7}\times\overline{\frac{n-8}{nr}\times\frac{7}{m}+\frac{n-9}{nr^2}\times\frac{7}{m}+\frac{n-10}{nr^3}\times\frac{7}{m}}$, &c. or to $\frac{1}{r^7}\times\frac{n-7}{n}\times\frac{m-7}{m}\times\frac{n-8}{\overline{n-7}\,r}\times\overline{\frac{1}{m-7}+\frac{n-9}{\overline{n-7}\,r^2}\times\frac{2}{m-7}+\frac{n-10}{\overline{n-7}\,r^3}\times\frac{3}{m-7}}$, &c. $+\frac{1}{r^7}\times\frac{7}{m}\times\frac{n-7}{n}\times\overline{\frac{n-8}{\overline{n-7}\,r}+\frac{n-9}{\overline{n-7}\,r^2}+\frac{n-10}{\overline{n-7}\,r^3}}$, &c. which is the very rule given for ſolving this queſtion, as will appear from notes (B) and (C).

Note (E). See the Scholium to Queſt. X.

ACCORDING to the calculations, the time in which the firſt yearly payment of a reverſionary *annuity* becomes due, is the end of the year in which the event happens that entitles to it, however little or much of the year may then happen to be unelapſed. And this, likewiſe, is the time when a reverſionary *ſum* becomes due. Thoſe who know how the calculations of the values of reverſions are inſtituted, muſt know this. But an annuity, the firſt payment of which is to be made at the ſame time with another payment of a ſum in hand, ſufficient to buy an equal annuity, is worth one year's purchaſe more than the ſum. For inſtance. Reckoning intereſt at 4 per cent. and r being 1*l.* increaſed by its intereſt for a year, or 1.04, $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$, &c. $= 25l.$ is the preſent value of an eſtate of 1*l.* per annum for ever. That is, it is the value of it, ſuppoſing the firſt rent of it is to be paid a year hence.——If the firſt rent is to be received immediately, or at the ſame time with another payment of 25*l.* it is worth one year's purchaſe more, or equivalent to 26*l.*——I have not found, that any of the writers on annuities and reverſions, have attended to this obſervation. It ſuggeſts a correction neceſſary to be applied to the common ſolutions of ſeveral important problems: particularly to the 21ſt and 22d in Mr. *Simpſon's Treatiſe on Annuities*, and the 26th, 27th, 32d, 33d, and 40th problems in his *Select Exerciſes*; and to all other problems of the ſame kind in other writers. There can be no great occaſion for being more explicit. It will not, however, be amiſs to add the following demonſtration: —— $\frac{1}{n}$ is the preſent probability that a life, whoſe complement is n, will fail in any one aſſignable year of its duration. $S\times\overline{\frac{1}{nr}+\frac{1}{nr^2}}$ $\overline{+\frac{1}{nr^3}}$, &c. (n), or the preſent value of 1*l.* per annum for n years, multiplied by $\frac{S}{n}$, is the preſent value of the ſum or legacy denoted by S, payable at the failure of the given life. Therefore, (n being 56; the life 30; intereſt 4 per cent. $r=1.04$; the ſum 25*l.*) the value of the expectation, by Mr. *De Moivre*'s hypotheſis, is 9.919.

Further. The value of 1*l.* to be received at the end of a year, provided the life whoſe complement is n fails, is the probability of the failure of the life multiplied by 1*l.* diſcounted for a year, or $\overline{1-\frac{n-1}{n}}\times\frac{1}{r}$. In like manner; the value of 1*l.* to be received at the end of 2 years, if the ſame life fails in 2 years, is $\overline{1-\frac{n-2}{n}}\times\frac{1}{r^2}$. And, therefore, the value of all the *poſſible* payments of an eſtate or annuity of 1*l.* for ever, to be entered upon after the given life, is $\overline{1-\frac{n-1}{n}}\times\frac{1}{r}+\overline{1-}$ $\overline{\frac{n-2}{n}}\times\frac{1}{r^2}+\overline{1-\frac{n-3}{n}}\times\frac{1}{r^3}$, &c. $(n)+\frac{1}{r^{n+1}}+$ $\frac{1}{r^{\ +2}}$

$\frac{1}{r^{n+2}}$, &c. or $\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$, &c. $- \overline{\frac{n-1}{nr} + \frac{n-2}{nr^2} +}$ $\overline{\frac{n-3}{nr^3}}$, &c. that is, the value of the life ſubtracted from the perpetuity; or, in this example, 14*l.* 684, (the value of a life at 30) ſubtracted from 25; that is, 10*l.* 316. But 10.316 is to 9.919, in the ſame ratio with 104 to 100, or 26 to 25, agreeably to the rule in the *Scholium.*

Note

Note (F.) Queſtion XIII. Page 44.

WHEN I here call 48 the mean age of all married men, and 40 the mean age of married women, I do not intend to ſuppoſe, that there are as many married perſons who exceed theſe ages, as there are who fall ſhort of them. It is likely that the latter are moſt numerous; and it is neceſſary that this ſhould be the caſe, to render the ſuppoſition I make juſt——If all marriages commenced at 33 for the man, and 25 for the woman, one half of them would be diſſolved by the time the men were 50, and the women 42; for (by the *Hypotheſis*, and alſo nearly by the *Breſlaw*, *Norwich*, and *Northampton* tables) there is an equal chance for the joint continuance of two lives, whoſe ages are 25 and 33, *ſeventeen* years. Forty-two and fifty then would be properly the mean ages at which widowhood would commence; meaning by theſe, "the "ages on each ſide of which equal numbers are "left widows and widowers."——But, tho' in this caſe half the marriages of every year would be diſſolved in 17 years, they would not be *all* diſſolved in twice that time. So far would this be from happening, that about a 7th part would continue beyond twice 17 years; nor would it be *certain*, that they would be all diſſolved till near the extremity of the poſſible extent of life. Tho', therefore, an equal number of marriages would be diſſolved, or an equal number of widows and widowers left *before* 50 and 42, and *afterwards*, yet the ages of the latter would, one with another, much more exceed 50 and 42, than the ages of the former (that is, of the widows and widowers left *before* 50 and 42) would fall ſhort of them. And the number of marriages alſo in the world, among perſons of greater ages than theſe, would be much fewer than among perſons of leſſer ages——In other words: the period, at which the marriages that have been contracted are half diſſolved, is not the period at which the number of marriages conſtantly exiſting is equally divided, but this period falls ſome years ſooner; and the period I have in view, falls in that part of the interval between theſe two periods, where the greater ages of the marriages on one ſide, are juſt enough to compenſate (in ſuch a calculation as that I have given) their deficiencies in number, compared with the number of marriages on the other ſide.

In ſhort. Suppoſe 35 marriages every year, between perſons 33 and 25. In 12 years there would be half as many in the world, as could poſſibly ariſe from ſuch a number of yearly weddings. In 17 years, half every ſet would be extinct. The *expectation* of every marriage would be 19 years, by prob. 21 of Mr. *De Moivre's Treatiſe on Annuities*, or by the note p. 299: that is, taking them all together, they would exiſt juſt as long as an equal number of *ſingle* perſons, ſuppoſed to be ſure of living juſt 19 years, and no more; or, as long as an equal number of ſingle perſons, all 48 years of age, ſuppoſed to be ſubject to the common laws of mortality. One with another, then, they will be all extinct in 19 years: the marriages which continue beyond this term, tho' fewer in number, enjoying among them juſt as much *more* duration, as thoſe that fall ſhort of it enjoy *leſs*. *Widows*, then, at a medium, will commence widowhood at 44 (that is, 25 increaſed by 19) years of age, and *widowers* at 52. The values, therefore, of the lives of the *former*, when they commence widowhood, will, one with another, be the ſame with the value of a life at 44; or, (reckoning intereſt at 4 per cent.) 12.5 years purchaſe, in preſent payment, (the annuity to begin at the end of a year); and their *expectations* of life will be 21 years, or half the difference between 44 and 86. The value of the lives of the *latter* will be 10.92, and their *expectation* 17 years—The whole number of marriages conſtantly exiſting, which would reſult from 35 ſuppoſed to commence annually, would be 19×35, or 665; and 53 years (the difference between 33 and 86) would be the time in which they would increaſe to this number——The chance of ſurvivorſhip would be 69 to 53, by prob. 18th, Mr. *De Moivre on Annuities*; that is, in 53 years, 35 relicts of theſe marriages would be left every year, and the number of *widows* would be to the number of *widowers*, as 69 to 53; or 19.8 *widows* would be left annually, and 15.2 *widowers*. The *maximum* of widows in life together, if none married, would be 21×19.8, or 416; and they would increaſe to this number in 114 years (or 61 years after the number of marriages had attained to a *maximum*)——The *maximum* of *widowers* would be 15.2×17, or 258; and they would increaſe to this number in 106 years.

An eaſy method may be hence deduced of ſolving the queſtion which occaſions this note——If the number of the members of the eſtabliſhment I have ſuppoſed, is 665, and the mean ages at which marriage may be deemed to commence are 25 and 33, 19.8 widows will (it has juſt appeared) be left every year; and the values of their lives, when they commence widowhood, will be, one with another, $12\frac{1}{2}$ years purchaſe. An annuity of 20*l.* will, therefore, be worth, to each widow, 250*l.* and 19.8 ſuch annuities muſt be worth 4950*l.* which, conſequently, is the annual income neceſſary for the ſupport of the eſtabliſhment, the firſt payment to be received immediately: or *l.* 7.44 from each of the 665 members; which anſwers nearly to the determination in page 44.

In the Laſt Eſſay, p. 296, it has been ſhewn, that obſervations determine the chance of ſurvivorſhip in favour of the wife in marriage, to be really ſo great as 3 to 2; and in ſome circumſtances greater. I have alſo there obſerved, that in order to account for this, from the difference of age between men and their wives, this difference muſt be at leaſt 12 years, and the mean ages of all who marry annually, muſt be ſuppoſed to be about 23 and 35. In this caſe, 19, as before, will nearly be the *expectation* of all marriages. The mean age at which widows and widowers will commence ſuch will be 42 and 54. The number of annual marriages neceſſary to keep up 665 marriages conſtantly exiſting, will be 35. The number of widows left annually, by ſuch a number of marriages, will be 21; and the values of their lives, at the time they commence widowhood, will be 12.85 years purchaſe by Table VI: and therefore, the whole annual income neceſſary for the ſupport of the ſuppoſed eſtabliſhment, will be 5397*l.* or an annual payment, beginning immediately, of *l.* 8.11 from each member—The number of widows on ſuch an eſtabliſhment will, in 63 years, grow, if none marry, to 462; and the number of widowers to 224.——It may be depended on, that all this would happen as far as Dr. *Halley*'s table, or the tables for *Norwich* and *Northampton*, exhibit the true ſtate of human mortality.

Among the miniſters and profeſſors in SCOTLAND, the number of married men being 667, or nearly that here mentioned, the number of annual weddings has, for many years, been at an average 32, and the number of widows left annually 20; and, therefore, the chance of ſurvivorſhip in favour of the wife, as 20 to 12, or 5 to 3. See Eſſay IV. p. 269. This is not more different, from the reſults I have given, than might have been expected; and the chief reaſon of the difference is, that the *expectations* of *ſingle* and *joint* lives among the miniſters and their wives in SCOTLAND, are greater than thoſe given by Dr. *Halley*'s, and the other tables of obſervation——Theſe tables give the expectations of lives as they are among the bulk of mankind in moderate towns. The expectations of lives among the better ſort of men, living moſtly in country villages and pariſhes, are much greater. The fact is, that among the miniſters in *Scotland*, the expectation of a *ſingle* life at the age of 27, is three years and an half greater; and, conſequently, of *joint* lives, above two years greater, than the ſame expectations by Dr. *Halley*'s Table. Ibid, page 264.

I cannot help juſt mentioning another remark here.——It may be obſerved, that ſuppoſing no ſecond marriages, and, at the ſame time, that the odds for the woman's ſurviving in marriage is 3 to 2, the number of *widows* in the world would be *double* the number of *widowers*. But it has been found, in fact, that the number of widows is five times the number of widowers. How this is to be accounted for, I have ſhewn in the Eſſay juſt referred to, page 270, 271.

Note (G). Queſtion XIV. Page 48.

LET r be 1*l.* increaſed by its intereſt for one year; t the given time or number of years for which the aſſurance is to be made; a, b, c, &c. the *probabilities* taken out of a table of obſervations, that the perſon whoſe age is given ſhall live 1, 2, 3, &c. years; and P the probability that he ſhall live t years. Then $\frac{1-a}{r} + \frac{1-b}{r^2} + \frac{1-c}{r^3}$, &c. $(t-1) + \frac{1-P}{r^t} + \frac{1-P}{r^{t+1}} + \frac{1-P}{r^{t+2}}$&c. $= \frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}$, &c. $(t) - \overline{\frac{a}{r} + \frac{b}{r^2} + \frac{c}{r^3}, \text{\&c.}\ (t-1) + \frac{P}{r^t}} + \frac{1-P}{r^t} \times \overline{\frac{1}{r} + \frac{1}{r^2} + \frac{1}{r^3}}$, &c. will be the exact value of an annuity to be entered upon at the failure of the given life, provided it happens in t years. And the rule is nothing but this value expreſſed in words. In a ſimilar manner may be demonſtrated the other rule for finding the values of aſſurances, for a given time, on two joint lives, or the longeſt of two lives.

Note (H). Queſtion XV. page 56.

LET r ſignify as before; S the given ſum to be aſſured; t the given time; N and n the number of the living in the table of obſervations, at the age of A and B reſpectively; A, B, C, &c. and a, b, c, &c. the number of the living in the table, at the end of 1, 2, 3, &c. years from the ages of A and B; D, D_{I}, D_{II}, D_{III}, &c. and d, d_{I}, d_{II}, d_{III}, &c. the decrements of life in the table, at the end of 1, 2, 3, &c. years from the ſame ages. Then, by reaſoning in the ſame manner with Mr. *Simpſon*, in p. 316, &c. *Select Exerciſes*, it will appear that

$$\mathrm{S} \times \overline{\frac{\mathrm{A} \times d}{\mathrm{N}nr} + \frac{\mathrm{B} \times d_{\text{I}}}{\mathrm{N}nr^2} + \frac{\mathrm{C} \times d_{\text{II}}}{\mathrm{N}nr^3}, \text{\&c. } (t)} + \mathrm{S} \times \overline{\frac{\mathrm{D}d}{2\mathrm{N}nr} + \frac{\mathrm{D}_{\text{I}}d_{\text{I}}}{2\mathrm{N}nr^2} + \frac{\mathrm{D}_{\text{II}}d_{\text{II}}}{2\mathrm{N}nr^3}, \text{\&c. } (t)} = \frac{\mathrm{S}}{n} \times \overline{\frac{\mathrm{A}d}{\mathrm{N}r} + \frac{\mathrm{B}d_{\text{I}}}{\mathrm{N}r^2} + \frac{\mathrm{C}d_{\text{II}}}{\mathrm{N}r^3}, \text{\&c. } (t)} + \frac{\mathrm{S}}{2\mathrm{N}} \times \overline{\frac{\mathrm{D}d}{nr} + \frac{\mathrm{D}_{\text{I}}d_{\text{I}}}{nr^2}, \text{\&c. } (t)}.$$

This is the exact anſwer to Queſtion XV. and the rule is as near an aproximation to it as there is reaſon to deſire.

In the ſame manner, retaining all the ſame ſymbols, it may be found, that the anſwer to Queſtion XVI. is

$$\mathrm{S} \times \overline{\frac{\mathrm{D}d}{2\mathrm{N}nr} + \frac{\mathrm{D}d_{\text{I}}}{\mathrm{N}nr^2} + \frac{\overline{\mathrm{D}+\mathrm{D}_{\text{I}}} \times d_{\text{II}}}{\mathrm{N}nr^3} + \frac{\overline{\mathrm{D}+\mathrm{D}_{\text{I}}+\mathrm{D}_{\text{II}}} \times d_{\text{III}}}{\mathrm{N}nr^4}\ (t), \text{\&c.}} + \mathrm{S} \times \overline{\frac{\mathrm{D}_{\text{I}}d_{\text{I}}}{2\mathrm{N}nr^2} + \frac{\mathrm{D}_{\text{II}}d_{\text{II}}}{2\mathrm{N}nr^3} + \frac{\mathrm{D}_{\text{III}}d_{\text{III}}}{2\mathrm{N}nr^4}, \text{\&c.}}$$

$(t-1)$

$$(t-\text{1}) = \frac{S}{nr} \times \overline{\frac{Dd_{\text{I}}}{Nr} + \frac{\overline{D+D_{\text{I}}} \times d_{\text{II}}}{Nr^2} + \frac{\overline{D+D_{\text{I}}+D_{\text{II}}} \times d_{\text{III}}}{Nr^3}},$$

$$\text{\&c. } (t-\text{1}) + \frac{S}{2N} \times \overline{\frac{Dd}{nr} + \frac{D_{\text{I}}d_{\text{I}}}{nr^2} + \frac{D_{\text{II}}d_{\text{II}}}{nr^3}}, \text{ \&c. } (t).$$

But $\frac{D}{Nr} + \frac{D+D_{\text{I}}}{Nr^2} + \frac{D+D_{\text{I}}+D_{\text{II}}}{Nr^3}$, &c. $(t-\text{1})$ is the ſame with the exceſs of the value of an annuity *certain* for a number of years leſs by one year than the given term, above the value of an annuity on the life of A, for the ſame number of years; from whence the reaſon of the rule for ſolving this queſtion, may be eaſily diſcovered.

Note (I). Page 119, &c.

LET t be any given term of years; p the value of 1*l.* due at the end of the given term; A the value of an annuity certain for the ſame term; n the *complement* of a given life; G the value for the given term, of two joint lives, both equal to the given life; (to be found by Queſt. VI.) P the perpetuity; r, 1*l.* increaſed by its intereſt for one year.

Then $\overline{A-G}\times n+t\times p\times P-A\times P\times r$ will be the preſent value of 1*l.* 2*l.* 3*l.* &c. (t) payable at the end of 1, 2, 3, &c. (t) years; but ſubject to failure when the given life fails.

If ſuch a courſe of payment is to begin immediately, and to be made at the beginning of every year, till $t+1$ payments are made in t years; add to the preceding value, the value increaſed by unity of an annuity on the given life for t years, found by Queſtion VI. and the *ſum* will be the value ſought. And this value, divided by the preſent value of what may happen to remain of the given life after t years, found by Queſtion VI. will give the *ſtanding annuity* to which ſuch a ſeries of increaſing annual payments, beginning immediately, will entitle, for the remainder of the given life after t years.

With the aſſiſtance of this theorem, all that is ſaid in page 119, &c. may be inveſtigated. It would be too tedious to enter into a more minute account.

Note (K). Page 149.

LET d ſignify the *difference* between the *complements* of the youngeſt and the oldeſt life in the body of Annuitants, here deſcribed, at the time they enter; let S ſignify the ſum of theſe *complements*; n any given number of years not greater than $\frac{S}{2} - \frac{d}{2}$; and x the ratio of the whole number of Annuitants to $\frac{S \times d}{2}$. Then

$x \times d$ will be the number that will die the 1ſt year;

$x \times \overline{d + \frac{2d}{S}}$ the number that will die the 2d year;

$x \times \overline{d + \frac{4d}{S} + \frac{4d}{S^2}}$, 3d year;

$x \times \overline{d + \frac{6d}{S} + \frac{8d}{S^2} + \frac{8d}{S^3}}$, 4th year;

$x \times \overline{d + \frac{8d}{S} + \frac{12d}{S^2} + \frac{16d}{S^3} + \frac{16d}{S^4}}$, 5th year;

and $x \times \overline{nd + \overline{n^2 - n} \times \frac{d}{S} + \overline{n - 2 + \overline{n - 2}\rceil^2 \times \frac{2d}{S^2}} + \overline{\overline{n - 3}}}$

$\overline{+ \overline{\overline{n - 3}\rceil^2 \times \frac{4d}{S^3}} + \overline{\overline{n - 4} + \overline{n - 4}\rceil^2 \times \frac{8d}{S^4}}}$, &c. (n)

will be the whole number dying in n years. When n is greater than $\frac{S}{2} - \frac{d}{2}$, this ſeries is greater than the whole number dying in n years; but in all other caſes it gives this number exactly, ſuppoſing the probabilities of life to decreaſe uniformly.——

In the preſent inſtance, the youngeſt life being 30, and the oldeſt 60, the two complements are 56 and 26. $S = 82$. $d = 30$. $\frac{Sd}{2} = 1230$. And therefore $x = \frac{33,333}{1230} = 27.1$. Take $n = 30$ years, and the foregoing ſeries will be $27.1 \times \overline{900 + 318.2 + 7.242 + .164} = 33.214$, which is a little greater than the whole number dying in 30 years, but at the ſame time leſs than the whole number of Annuitants.

Note

Note (L). See Eſſay I. Page 169, 170, 172.

THE *ſum* of the probabilities that any given lives will attain to the end of the 1ſt, 2d, 3d, &c. *years* from the preſent time to the utmoſt extremity of life (for inſtance, $\frac{45}{46} + \frac{44}{46} + \frac{43}{46}$, &c. to $\frac{1}{46} = 22\frac{1}{2}$ for lives of 40, by the *hypotheſis*) may be called their *expectation*, or the number of payments due to them, as *yearly annuitants*. The ſum of the probabilities that they will attain to the end of the 1ſt, 2d, 3d, &c. *half years* (or, in the particular caſe ſpecified, $\frac{91}{92} + \frac{90}{92} + \frac{89}{92} + \frac{88}{92}$, &c. $=$ $\frac{91}{2}$ *half* years, or $22\frac{3}{4}$ *years*) is their expectation as *half yearly annuitants*. And the ſums juſt mentioned of the probabilities of their attaining to the end of the 1ſt, 2d, 3d, &c. *moments* (equal in the ſame particular caſe to 23 years) is properly their *expectation of life*, or their *expectation* as annuitants ſecured by land.

Mr. *De Moivre* has omitted the demonſtrations of the rules he has given for finding the *expectations* of lives, and only intimated, in general, that he diſcovered them by a calculation deduced from the method of fluxions. See his *Treatiſe on Annuities*, page 66. It will, perhaps, be agreeable to ſome to ſee how eaſily they are deduced in this method, upon the hypotheſis of an equal decrement of life.

Let $\dot{x}$ ſtand for a moment of time, and n the *complement* of any aſſigned life. Then $\frac{n-\dot{x}}{n}$, $\frac{n-2\dot{x}}{n}$, $\frac{n-3\dot{x}}{n}$, &c. will be the *preſent* probabilities of its continuing to the end of the 1ſt, 2d, 3d, &c. moments; and $\frac{n-x}{n}$ the probability of its continuing to the end of x time. $\frac{n-x}{n} \times \dot{x}$ will therefore be the *fluxion* of the ſum of the probabilities, or of an *area* repreſenting this ſum, whoſe *ordinates* are $\frac{n-x}{n}$, and *axis* x.—The *fluent* of this expreſſion, or $x - \frac{x^2}{2n}$, is the ſum itſelf for the time x; and this, when $x = n$, becomes $\frac{1}{2}n$, and gives the *expectation* of the aſſigned life, or the ſum of all the probabilities juſt mentioned for its whole poſſible duration.——In like manner: ſince $\frac{\overline{n-x}^2}{n^2}$ is the probability that two equal joint lives will continue x time, $\frac{\overline{n-x}^2}{n^2} \times \dot{x}$ will be the *fluxion* of the ſum of the probabilities. The *fluent* is $x - \frac{x^2}{n} + \frac{x^3}{3n^2}$, which, when $n = x$, is $\frac{n}{3}$, or the expectation of two equal joint lives.——Again: ſince $\frac{n-x}{n} \times \frac{2x}{n}$ is the probability that there will be a ſurvivor of two equal joint lives at the end of x time, $\frac{n-x}{n} \times \frac{2x}{n} \times \dot{x}$ will be the *fluxion* of the ſum of the probabilities; and the *fluent*, or $\frac{x^2}{n} - \frac{2x^3}{3x^2}$ is (when $x = n$) $\frac{1}{3}n$, or the *expectation* of ſurvivorſhip between two equal lives; which, therefore, appears to be equal to the *expectation* of their joint continnance. The expectation of two *unequal* joint lives, found in the ſame way, is $\frac{m}{2} - \frac{m^2}{6n}$, *m* being the *complement* of the oldeſt life, and *n* the *complement* of the youngeſt. The whole expectation of ſurvivorſhip is $\frac{n}{2} - \frac{m}{2} + \frac{m^2}{3n}$. The expectation of ſurvivorſhip, on the part of the oldeſt is $\frac{m^2}{6n}$; and the expectation, on the part of the youngeſt, is $\frac{n}{2} - \frac{m}{2} + \frac{m^2}{6n}$. It is eaſy to apply this inveſtigation to any number of joint lives, and to all caſes of ſurvivorſhip.

It may be obſerved, concerning the firſt of the fluents here given, that it expreſſes not only the expectation of a given life for the time *x*, and therefore its whole expectation when $x = n$, but likewiſe, the number of perſons alive, to which one perſon added annually to a ſociety, at a given age, will increaſe in *x* time.——Thus: Suppoſe one annuitant, whoſe age is 28, (and whoſe *complement* of life, therefore, is 58, or *expectation* of life 29) to come upon a ſociety every year; the number of annuitants alive, deduced from hence, will, in *x* years, be $x - \frac{x^2}{4 \times 29}$, or $\frac{4 \times 29 - x^2}{4 + 29} \times x$; and, therefore, the number of annuitants alive, deduced in the ſame time from *p* annuitants left annually at the ſame age, will be $\frac{4 \times 29 - x^2}{4 \times 29} \times px$.——In like manner, the 2d fluent, or $\frac{x^3}{3n^2} - \frac{x^3}{n} + x$, gives the number of marriages in being together, that will, in x years, grow out of *one* yearly marriage, between perſons of *equal* ages, whoſe complement of life is n. If they are of *unequal* ages, and the complement of the oldeſt life is m, and of the youngeſt n, this number will be $\frac{x^3}{3nm} - \frac{\overline{n+m}\times x^2}{2nm} + x$. And if the number of years is required, in which any given number of yearly marriages, between men and women at given ages, will increaſe ſo far as to be in any given proportion to the greateſt number that can poſſibly grow out of ſuch marriages, this expreſſion muſt be made equal to the *expectation* of the joint lives, or of each marriage, multiplied by the fraction expreſſing the given proportion; and the root of the equation will be the anſwer. Thus: it may be found, that one marriage every year, between perſons 33 and 25 years of age, would in 10 years increaſe to 8.35; in 15 years, to 11.38; and in 53 years, to 19, or their greateſt poſſible number: and, conſequently, that 35 ſuch yearly marriages would, in 10 years, increaſe to 292; in 15 years, to 398; and in 53 years, to 665.——And if it is enquired in what number of years 35 ſuch yearly marriages would increaſe to half the number in being together, poſſible to be derived from them, the value of x, in the cubic equation $\frac{x^3}{3nm} - \frac{\overline{n+m}\times x^2}{2nm} + x = \overline{\frac{m}{2} - \frac{m}{6n}} \times \frac{1}{2}$, muſt be found; which, in the preſent inſtance, is nearly 12.

I have, in ſome parts of this work, had occaſion to make ſuch deductions as theſe. See note (A), p. 277; and note (F), p. 288; and Queſtions III, and XIII.

Note (M). Eſſay II. Page 226.

LET r ſignify 1*l*, increaſed by its intereſt for one year.

V the PERPETUITY.

n the difference between the age of the youngeſt life, and 86; or its *complement.*

m the complement of the oldeſt life.

P the value (in Table II.) of an annuity certain for m years.

And the exact value of any two given joint lives, according to the hypotheſis of an equal decrement of life, will be $V - \frac{V+1}{n} \times \overline{\overline{n - m - 2v - 1} \times \frac{P}{m} + 2v}$. Example:

Let the ages be 27 and 38; and the rate of intereſt 4 *per cent.* Then $n = 59$. $m = 48$. $V = 25$. $P = 21.195$. $n - m - 2v - 1 = -40$. $\overline{\overline{n - m - 2v - 1} \times \frac{P}{m} + 2v} = 50 - 17.660 = 32.340$. And $V - \frac{V+1}{n} \times \overline{\overline{n - m - 2v - 1} \times \frac{P}{m} + 2v} = 25 - \frac{26}{59} \times 32.340 = 10.748$, the value of two joint lives whoſe ages are 27 and 38.

Note (N). Eſſay III. Page 232.

IT is plain, that the purchaſer of A's right, as ſtated in the firſt of the queſtions, to which this note refers, cannot get into poſſeſſion, till the year when A and B ſhall be both dead; nor then, unleſs A happens to die *laſt*. Now, ſuppoſing the common complement of life n; the probability that A and B ſhall be *both* dead at the end of the *firſt* year, and A die laſt, is $\overline{1-\frac{n-1}{n}}\times\overline{1-\frac{n-1}{n}}\times\frac{1}{2}=\frac{1}{2}-\frac{n-1}{2n}-\frac{n-1}{2n}+\frac{\overline{n-1}|^2}{2n^2}$.—— In like manner, the probability that they ſhall be *both* dead at the end of the 2d, 3d, &c. years, and A ſurvive, is $\frac{1}{2}-\frac{n-2}{2n}-\frac{n-2}{2n}+\frac{\overline{n-2}|^2}{2n^2}$; $\frac{1}{2}-\frac{n-3}{2n}-\frac{n-3}{2n}+\frac{\overline{n-3}|^2}{2n^2}$, &c. The *preſent* value, therefore, of the 1ſt, 2d, 3d, &c. rents of the reverſionary eſtate is $\frac{1}{2r}-\frac{n-1}{2nr}-\frac{n-1}{2nr}+\frac{\overline{n-1}|^2}{2nr}$, $\frac{1}{2r^2}-\frac{n-2}{2nr^2}-\frac{n-2}{2nr^2}+\frac{\overline{n-2}|^2}{2n^2r^2}$, $\frac{1}{2r^2}-\frac{n-3}{2nr^3}-\frac{n-3}{2nr^3}+\frac{\overline{n-3}|^2}{2n^2r^3}$, &c. Suppoſing r to ſignify 1*l.* increaſed by its intereſt for a year; and the eſtate to be 1*l.* *per annum*. And the *ſum* of theſe terms continued *in infinitum*, is the value *required*.——But $\frac{1}{2r}+\frac{1}{2r^2}+\frac{1}{2r^3}$, &c. is *half* the perpetuity. And $\frac{n-1}{2nr}+\frac{n-1}{2nr}-\frac{\overline{n-1}|^2}{2n^2r}+\frac{n-2}{2nr^2}+\frac{n-2}{2nr^2}-\frac{\overline{n-2}|^2}{2n^2r^2}+\frac{n-3}{2nr^3}+\frac{n-3}{2nr^3}-\frac{\overline{n-3}|^2}{2n^2r^3}$, &c. is *half* the value of the *joint* lives, ſubtracted from *half* the ſum of the values of the two *ſingle* lives; that is, *half* the value of the *longeſt* of the two lives.

A ſimilar demonſtration may be applied to the other queſtion.

Note (O). Eſſay IV. Page 262.

LET r be 1*l.* increaſed by its intereſt for one year.

Let S repreſent any given interval of time, or number of years, during which the decrements of life in a table of obſervations continue equal.

a the number of the living in the table at the beginning of the firſt year of that interval.

b the number of the living in the table at the beginning of the year immediately following the ſame interval.

P the value of an annuity certain for S years.

p the value, in Table I. of 1*l.* due at the end of S years.

Q the value, in Table VI. of an annuity for the life of a perſon whoſe age wants S years of 86.

N the value, in ſtrict agreement with the given table of obſervations, of an annuity on the life of a perſon whoſe age is S years greater than the age at which the interval of equal decrements begins. Then

$Q + \frac{b}{a} \times \overline{P - Q}$ will be the value, according to the table of obſervations, of an annuity for S years, on a life of the ſame age with that at which the interval of equal decrements begins. And

$Q + \frac{b}{a} \times \overline{P - Q + pN}$ will be the value of an annuity on the whole duration of that life.

When S repreſents *one* year, Q vaniſhes, and the laſt expreſſion becomes $\frac{b}{ar} \times \overline{1 + N}$; which is the rule for finding, from the value given of any life, the value of a life one year younger.

Theſe Theorems ſave much labour in calculating the values of life-annuities from tables of obſervations.

The firſt of them, with its inveſtigation, may be found in page 341, 3d edition, of Mr. *De Moivre*'s *Treatiſe on the Doctrine of Chances.* But it is neceſſary to obſerve, that the direction Mr. *De Moivre* has given for finding the value of Q is wrong. In conſequence of calculating agreeably to this direction, he gives the value of a life at the age of 42, by Dr. *Halley*'s table, greater than the value of the ſame life by his own hypotheſis; whereas, it is evident, that the probabilities of living after 42, being all along leſs in Dr. *Halley*'s table, than in the hypotheſis, the *value* of the life muſt be alſo leſs.

The mathematical reader may eaſily ſatisfy himſelf, that the value of Q ought to be taken from Table VI. as I have directed.

An eaſy and accurate method of finding the values of ſingles lives, agreeably to any given table of obſervations, is given by Mr. *Dodſon* in his *Mathematical Repoſitory*, vol. II. page 161.

There is alſo in Mr. *Simpſon's Select Exerciſes*, page 275, a very eaſy rule for approximating to the values of ſingle lives, according to Dr. *Halley*'s table. But this rule muſt not be depended on; for I have found it $\frac{1}{2}$ a year's purchaſe, and ſometimes three-quarters of a year's purchaſe wrong.

To prevent the danger of miſtaking the Theorem I have given, I have thought proper to ſubjoin the following example.

Let the table of obſervations be the *Breſlaw* Table, or Table III. The value of a life at 78, by this

Table, is $\frac{49}{58r}+\frac{41}{58r^2}+\frac{34}{58r^3}$, &c. to the end of life. The number of terms in this ſeries being ſmall, it may be eaſily found to be 3.514, ſuppoſing intereſt at 4 *per cent.* and $\frac{1}{r}$, $\frac{1}{r^2}$, $\frac{1}{r^3}$, &c. being the values, in Table I. of 1*l.* at the end of 1, 2, 3, &c. years.——From 78 to 74 the decrements of life continue equal; and therefore $S=4$. $a=98$. $b=58$. $P=3.6298$, by Table II; $p=.8548$, by Table I; $Q=1.406$, by Table VI; $N=3.514$. $P-Q+pN=5.227$; and $Q+\frac{b}{a}\times\overline{P-Q+pN}=4.500$, or the value of a life at 74.

From 74 to 70 there is another interval of equal decrements; and, by a like eaſy operation, the value of a life at 70 will be found to be 5.595.

TABLE I.

The preſent Value of 1*l.* to be received at the end of any number of years, not exceeding 100; diſcounting at the rates of 3, 3½, 4, 4½, 5 and 6 *per cent.* compound intereſt.

	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
1	,970874	,966184	,961538	,956938	.952381	,943396
2	,942596	,933511	,924556	,915730	,907029	,889996
3	,915142	,901943	,888996	,876297	,863838	,839619
4	,888487	,871442	,854804	,838561	,822702	,792094
5	,862609	,841973	,821927	,802451	,783526	,747258
6	,837484	,813501	,790315	,767896	,746215	,704961
7	,813092	,785991	,759918	,734828	.710681	,665057
8	,789409	,759412	,730690	,703185	,676839	,627412
9	,766417	,733731	.702587	,672904	,644609	,591898
10	,744094	,708919	,675564	,643928	,613913	,558395
11	,722421	,684946	,649581	,616199	,584679	.526788
12	,701380	,661783	.624597	,589664	,556837	,496969
13	,680951	,639404	,600574	,564272	,530321	,468839
14	,661118	,617782	,577475	,539973	,505068	,442301
15	,641862	,596891	,555265	,516720	,481017	,417265
16	,623167	,576706	,533908	,494469	,458112	.393646
17	,605016	,557204	,513373	.473176	,436297	,371364
18	,587395	,538361	,493628	,452800	,415521	,350344
19	,570286	,520156	,474642	,433302	,395734	.330513
20	,553676	,502566	,456387	,414643	.376889	,311805
21	,537549	,485571	,438834	,396787	,358942	,294155
22	,521893	,469151	,421955	,379701	341850	,277505
23	,506692	,453286	,405726	,363350	.325571	,261797
24	,491934	,437957	,390121	.347703	,310068	,246979
25	,477606	,423147	,375117	,332731	,295303	.232999
26	,463695	,408838	,360689	,318402	,281241	,219810
27	,450189	,395012	,346817	,304691	.267848	,207368
28	,437077	,381654	.333477	,291571	,255094	,195630
29	,424346	,368748	,320651	,279015	,242946	,184557
30	,411987	.356278	,308319	,267000	,231377	,174110
31	,399987	,344230	,296460	,255502	,220359	,164255
32	.388337	.332590	.285058	,244500	.209866	,154957

TABLE I. Continued.

	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
33	377026	,321343	,274094	,233971	,199873	,146186
34	,366045	,310476	,263552	,223896	,190355	,137912
35	,355383	,299977	,253415	,214254	,181290	,130105
36	,345032	,289833	,243669	,205028	,172657	,122741
37	,334983	,280032	,234297	,196199	,164436	,115793
38	,325226	,270562	,225285	,187750	,156605	,109239
39	,315754	,261413	,216621	,179665	,149148	,103056
40	,306557	,252572	,208289	,171929	,142046	,097222
41	,297628	,244031	,200278	,164525	,135282	,091719
42	,288959	,235779	,192575	,157440	,128840	,086527
43	,280543	,227806	,185168	,150663	,122704	,081630
44	,272372	,220102	,178046	,144173	,116864	,077009
45	,264439	,212659	,171198	,137964	,111297	,072650
46	,256737	,205468	,164614	,132023	,105997	,068538
47	,249259	,198520	,158283	,126338	,100949	,064658
48	,241999	,191806	,152195	,120898	,096142	;060998
49	,234950	,185320	,146341	,115692	,091564	,057546
50	,228107	,179053	,140713	,110710	,087204	,054288
51	,221463	,172998	,135301	,105942	,083051	,051215
52	,215013	,167148	,130097	,101380	,079096	,048316
53	,208750	,161496	,125093	,097014	,075330	,045582
54	,202670	,156035	,120282	,092837	,071743	,043001
55	,196767	,150758	,115656	,088839	,068326	,040567
56	,191036	,145660	,111207	,085013	,065073	,038271
57	,185472	,140734	,106930	,081353	,061974	,036105
58	,180070	,135975	,102817	,077849	,059023	,034061
59	,174825	,131377	,098963	,074497	,056212	,032133
60	,169733	,126934	,095060	,071289	,053536	,030310
61	,164789	,122642	,091404	,068219	,050986	,028598
62	,159990	,118495	,087889	,065281	,048558	,026989
63	,155330	,114487	.084508	,062470	,046246	,025453
64	,150806	,110616	,081258	,059780	,044044	,024012
65	.146413	,106875	,078133	,057206	,041946	,022653
66	,142149	,103261	,075128	,054742	,039949	,021370
67	,138009	,099769	,072238	,052385	,038047	,020161
68	,133989	,096395	.069460	,050129	,036235	,019020
69	,130086	.093136	.066788	,047971	,034509	.017943
70	,126297	,019986	,064219	,045905	,032866	016927

TABLE I. Continued.

	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 par Ct.	6 per Ct.
71	,122619	,086943	,061749	,043928	,031301	,015969
72	,119047	,084003	,059374	,042037	,029811	,015065
73	,115580	,081162	,057091	,040226	,028391	,014212
74	,112214	,078418	,054895	,038494	,027039	,013408
75	,108945	,075766	,052784	,036836	,025752	,012649
76	,105772	,073204	,050754	,035250	,024525	,011933
77	,102691	,070728	,048801	,033732	,023357	,011258
78	,099700	,068336	,046924	,032280	,022245	,010620
79	,096796	,066026	,045120	,030890	,021186	,010019
80	,093977	,063793	,043384	,029559	,020177	,009452
81	,091240	,061636	.041716	,028287	,019216	,008917
82	,088582	,059551	,040111	,027068	,018301	,008412
83	,086002	.057538	,038569	,025903	,017430	,007936
84	,083497	,055592	,037085	,024787	,016600	,007487
85	,081065	,053712	,035659	,023720	,015809	,007063
86	,078704	,051896	,034287	,022699	,015056	,006663
87	,076412	,050141	,032968	,021721	,014339	,006286
88	,074186	,048445	,031700	,020786	,013657	,005930
89	,072027	,046807	,030481	,019891	,013006	,005595
90	,069928	,045224	,029309	,019034	,012387	,005278
91	,067891	,043695	,028182	,018215	,011797	,004979
92	,065914	,042217	,027098	,017430	,011235	,004697
93	,063994	,040789	,026055	,016680	,010700	,004432
94	,062130	,039410	,025053	,015961	,010191	,004181
95	,060320	,038077	,024090	,015274	,009705	,003944
96	,058563	,036790	,023163	,014616	,009243	,003721
97	,056858	,035546	,022272	,013987	,008803	,003510
98	,055202	,034344	,021416	,013385	,008384	,003312
99	,053594	,033182	,020592	,012808	,007985	,003124
100	,052033	,032060	,019800	,012257	,007604	,002957

TABLE II.

The preſent Value of an Annuity of One Pound, for any Number of Years not exceeding 100, at the ſeveral Rates of 3, 3½, 4, 5, and *6l. per Cent.*

Ye.	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
1	.9708	.9662	.9615	.9523	.9433
2	1.9133	1.8997	1.8860	1.8594	1.8333
3	2.8286	2.8016	2.7750	2.7232	2.6730
4	3.7170	3.6731	3.6298	3.5459	3.4651
5	4.5797	4.5151	4.4518	4.3294	4.2123
6	5.4971	5.3286	5.2421	5.0756	4.9173
7	6.2302	6.1145	6.0020	5.7863	5.5823
8	7.0196	6.8740	6.7327	6.4632	6.2097
9	7.7861	7.6077	7.4353	7.1078	6.8010
10	8.5302	8.3166	8.1108	7.7212	7.3600
11	9.2526	9.0015	8.7604	8.3064	7.8868
12	9.9540	9.6633	9.3850	8.8632	8.3838
13	10.6349	10.3027	9.9856	9.3935	8.8526
14	11.2960	10.9205	10.5631	9.8986	9.2949
15	11.9379	11.5174	11.1183	10.3796	9.7122
16	12.5611	12.0941	11.6522	10.8377	10.1058
17	13.1661	12.6513	12.1656	11.2740	10.4772
18	13.7535	13.1897	12.6592	11.6895	10.8276
19	14.3238	13.7098	13.1339	12.0853	11.1581
20	14.8774	14.2124	13.5903	12.4622	11.4699
21	15.4150	14.6980	14.0291	12.8211	11.7640
22	15.9389	15.1671	14.4511	13.1630	12.0415
23	16.4436	15.6204	14.8568	13.4885	12.3033
24	16.9355	16.0584	15.2469	13.7986	12.5503
25	17.4131	16.4815	15.6220	14.0939	12.7833

TABLE II. Continued.

Ye.	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
26	17.8768	16.8904	15.9827	14.3751	13.0031
27	18.3270	17.2854	16.3295	14.6430	13.2105
28	18.7641	17.6670	16.6630	14.8981	13.4061
29	19.1884	18.0358	16.9837	15.1410	13.5907
30	19.6004	18.3920	17.2920	15.3724	13.7648
31	20.0004	18.7363	17.5884	15.5928	13.9290
32	20.3887	19.0689	17.8735	15.8026	14.0840
33	20.7657	19.3902	18.1476	16.0025	14.2302
34	21.1318	19.7007	18.4111	16.1929	14.3681
35	21.4872	20.0007	18.6646	16.3741	14.4982
36	21.8322	20.2905	18.9082	16.5468	14.6209
37	22.1672	20.5705	19.1425	16.7112	14.7367
38	22.4924	20.8411	19.3678	16.8678	14.8460
39	22.8082	21.1025	19.5844	17.0170	14.9490
40	23.1147	21.3551	19.7927	17.1590	15.0462
41	23.4124	21.5991	19.9930	17.2943	15.1380
42	23.7013	21.8349	20.1856	17.4232	15.2245
43	23.9819	22.0627	20.3707	17.5459	15.3061
44	24.2542	22.2828	20.5488	17.6627	15.3831
45	24.5187	22.4955	20.7200	17.7740	15.4558
46	24.7754	22.7009	20.8846	17.8800	15.5243
47	25.0247	22.8994	21.0429	17.9810	15.5890
48	25.2667	23.0912	21.1951	18.0771	15.6500
49	25.5016	23.2766	21.3414	18.1687	15.7075
50	25.7297	23.4556	21.4821	18.2559	15.7618
51	25.9512	23.6286	21.6174	18.3389	15.8130
52	26.1662	23.7958	21.7475	18.4180	15.8613
53	26.3749	23.9573	21.8726	18.4934	15.9069
54	26.5776	24.1133	21.9929	18.5651	15.9499
55	26.7744	24.2641	22.1086	18.6334	15.9905

TABLE II. Continued.

Ye.	3 per Ct.	3½ per Ct.	4 per Ct.	5 per Ct.	6 per Ct.
56	26.9654	24.4097	22.2198	18.6985	16.0288
57	27.1509	24.5504	22.3267	18.7605	16.0649
58	27.3310	24.6864	22.4295	18.8195	16.0989
59	27.5058	24.8178	22.5284	18.8757	16.1311
60	27.6755	24.9447	22.6234	18.9292	16.1614
61	27.8403	25.0674	22.7148	18.9802	16.1900
62	28.0003	25.1859	22.8027	19.0288	16.2170
63	28.1556	25.3004	22.8872	19.0750	16.2424
64	28.3064	25.4110	22.9685	19.1191	16.2664
65	28.4528	25.5178	23.0466	19.1610	16.2891
66	28.5950	25.6211	23.1218	19.2010	16.3104
67	28.7330	25.7209	23.1940	19.2390	16.3306
68	28.8670	25.8173	23.2635	19.2753	16.3496
69	28.9971	25.9104	23.3302	19.3098	16.3676
70	29.1234	26.0004	23.3945	19.3426	16.3845
71	29.2460	26.0873	23.4562	19.3739	16.4005
72	29.3650	26.1713	23.5156	19.4037	16.4155
73	29.4806	26.2525	23.5727	19.4321	16.4297
74	29.5928	26.3309	23.6276	19.4592	16.4431
75	29.7018	26.4067	23.6804	19.4849	16.4558
76	29.8076	26.4799	23.7311	19.5094	16.4677
77	29.9102	26.5506	23.7799	19.5328	16.4790
78	30.0099	26.6190	23.8268	19.5550	16.4896
79	30.1067	26.6850	23.8720	19.5762	16.4996
80	30.2007	26.7488	23.9153	19.5964	16.5091
81	30 2920	26.8104	23.9571	19.6156	16.5180
82	30.3805	26.8700	23.9972	19.6339	16.5264
83	30.4665	26.9275	24.0357	19.6514	16.5343
84	30.5500	26.9831	24.0728	19.6680	16.5418
85	30.6311	27.0368	24.1085	19.6838	16.5489

TABLE II. Continued.

Ye.	3 per Ct.	3½ per Ct.	4 per C.	5 per Ct.	6 per Ct.
86	30.7098	27.0887	24.1428	19.6988	16.5556
87	30.7862	27.1388	24.1757	19.7132	16.5618
88	30.8604	27.1873	24.2074	19.7268	16.5678
89	30.9324	27.2341	24.2379	19.7398	16.5734
80	31.0024	27.2793	24.2672	19.7522	16.5786
91	31.0703	27.3230	24.2954	19.7640	16.5836
92	31.1362	27.3652	24.3225	19.7752	16.5883
93	31.2002	27.4060	24.3486	19.7859	16.5928
94	31.2623	27.4454	24.3736	19.7961	16.5969
95	31.3226	27.4835	24.3977	19.8058	16.6009
96	31.3812	27.5203	24.4209	19.8151	16.6046
97	31.4380	27.5558	24.4431	19.8239	16.6081
98	31.4932	27.5902	24.4646	19.8323	16.6114
99	31.5468	27.6234	24.4852	19.8403	16.6145
100	31.5989	27.6554	24.5050	19.8479	16.6175

TABLE III.

Shewing the Probabilities of the Duration of Life, as deduced by Dr. *Halley* from Obſervations on the Bills of Mortality of BRESLAW.

Ages	Perſons living.	Decr. of Life.	Ages.	Perſons living.	Decr. of Life.	Ages	Perſons living.	Decr. of Life
1	1000	145	31	523	8	61	232	10
2	855	57	32	515	8	62	222	10
3	798	38	33	507	8	63	212	10
4	760	28	34	499	9	64	202	10
5	732	22	35	490	9	65	192	10
6	710	18	36	481	9	66	182	10
7	692	12	37	472	9	67	172	10
8	680	10	38	463	9	68	162	10
9	670	9	39	454	9	69	152	10
10	661	8	40	445	9	70	142	11
11	653	7	41	436	9	71	131	11
12	646	6	42	427	10	72	120	11
13	640	6	43	417	10	73	109	11
14	634	6	44	407	10	74	98	10
15	628	6	45	397	10	75	88	10
16	622	6	46	387	10	76	78	10
17	616	6	47	377	10	77	68	10
18	610	6	48	367	10	78	58	9
19	604	6	49	357	11	79	49	8
20	598	6	50	346	11	80	41	7
21	592	6	51	335	11	81	34	6
22	586	7	52	324	11	82	28	5
23	579	6	53	313	11	83	23	4
24	573	6	54	302	10	84	19	4
25	567	7	55	292	10	85	15	4
26	560	7	56	282	10	86	11	3
27	553	7	57	272	10	87	8	3
28	546	7	58	262	10	88	5	2
29	539	8	59	252	10	89	3	2
30	531	8	60	242	10	90	1	1

TABLE IV.

Shewing the Probabilities of Life at NORTHAMPTON. See page 255, 256.

Ages.	Perſons living.	Decr. of Life.	Ages.	Perſons living.	Decr. of Life.	Ages.	Perſons living.	Decr. of Life.
0	1149	300	31	428	7	62	187	8
1	849	127	32	421	7	63	179	8
2	722	50	33	414	7	64	171	8
3	672	26	34	407	7	65	163	8
4	646	21	35	400	7	66	155	8
5	625	16	36	393	7	67	147	8
6	609	13	37	386	7	68	139	8
7	596	10	38	379	7	69	131	8
8	586	9	39	372	7	70	123	8
9	577	7	40	365	8	71	115	8
10	570	6	41	357	8	72	107	8
11	564	6	42	349	8	73	99	8
12	558	5	43	341	8	74	91	8
13	553	5	44	333	8	75	83	8
14	548	5	45	325	8	76	75	8
15	543	5	46	317	8	77	67	7
16	538	5	47	309	8	78	60	7
17	533	5	48	301	8	79	53	7
18	528	6	49	293	9	80	46	7
19	522	7	50	284	9	81	39	7
20	515	8	51	275	8	82	32	6
21	507	8	52	267	8	83	26	5
22	499	8	53	259	8	84	21	4
23	491	8	54	251	8	85	17	4
24	483	8	55	243	8	86	13	3
25	475	8	56	235	8	87	10	2
26	467	8	57	227	8	88	8	2
27	459	8	58	219	8	89	6	2
28	451	8	59	211	8	90	4	2
29	443	8	60	203	8	91	2	1
30	435	7	61	195	8	92	1	1

TABLE V.

Shewing the Probabilities of Life at NORWICH.

See page 256, 257.

Ages	Perſons living.	Decr. of Life.	Ages	Perſons living.	Decr. of Life.	Ages	Perſons living.	Decr. of Life.
0	1185	320	32	392	6	63	174	9
1	865	160	33	386	6	64	165	9
2	705	60	34	380	6	65	156	9
3	645	32	35	374	6	66	147	9
4	613	23	36	368	6	67	138	9
5	590	20	37	362	6	68	129	9
6	570	16	38	356	6	69	120	9
7	554	13	39	350	7	70	111	9
8	541	11	40	343	6	71	102	8
9	530	9	41	337	6	72	94	8
10	521	7	42	331	6	73	86	8
11	514	6	43	325	7	74	78	8
12	508	6	44	318	7	75	70	8
13	502	5	45	311	7	76	62	7
14	497	5	46	304	7	77	55	7
15	492	5	47	297	7	78	48	6
16	487	5	48	290	7	79	42	5
17	482	5	49	283	7	80	37	5
18	477	5	50	276	7	81	32	4
19	472	5	51	269	7	82	28	4
20	467	6	52	262	7	83	24	4
21	461	6	53	255	8	84	20	3
22	455	6	54	247	8	85	17	3
23	449	6	55	239	8	86	14	3
24	443	6	56	231	8	87	11	2
25	437	6	57	223	8	88	9	2
26	431	7	58	215	8	89	7	2
27	424	7	59	207	8	90	5	2
28	417	7	60	199	8	91	3	2
29	410	6	61	191	8	92	1	1
30	404	6	62	183	9	93	1	1
31	398	6						

TABLE VI. (*a*).

Shewing the preſent Values of an Annuity of 1*l.* on a ſingle life, according to Mr. *De Moivre*'s hypotheſis; and, therefore, nearly, according to the probabilities of life at BRESLAW, NORWICH, and NORTHAMPTON. See p. 2, and p. 262.

Age.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
8	19,736	18,160	16,791	15,595	14,544	12,790
9	19,868	18,269	16,882	15,672	14,607	12,839
10	19,868	18,269	16,882	15,672	14,607	12,839
11	19,736	18,160	16,791	15,595	14,544	12,790
12	19,604	18,049	16,698	15,517	14,480	12,741
13	19,469	17,937	16,604	15,437	14,412	12,691
14	19,331	17,823	16,508	15,356	14,342	12,639
15	19,192	17,707	16,410	15,273	14,271	12,586
16	19,050	17,588	16,311	15,189	14,197	12,532
17	18,905	17,467	16,209	15,102	14,123	12,476
18	18,759	17,344	16,105	15,015	14,047	12,419
19	18,610	17,220	15,999	14,923	13,970	12,361
20	18,458	17,093	15,891	14,831	13,891	12,301
21	18,305	16,963	15,781	14,737	13,810	12,239
22	18,148	16,830	15,669	14,641	13,727	12,177
23	17,990	16,696	15,554	14,543	13,642	12,112
24	17,827	16,559	15.437	14,442	13,555	12,045
25	17,664	16,419	15,318	14,340	13,466	11,978
26	17,497	16,277	15,197	14,235	13,375	11,908
27	17,327	16,133	15,073	14,128	13,282	11,837
28	17,154	15.985	14,946	14,018	13,186	11,763
29	16,979	15,835	14,816	13,905	13,088	11,688
30	16,800	15,682	14,684	13,791	12,988	11,610
31	16,620	15,526	14,549	13,673	12,855	11,530
32	16,436	15,367	14,411	13,553	12,780	11,449
33	16,248	15,204	14,270	13,430	12,673	11.365

(*a*) This Table is the ſame with Mr. *De Moivre*'s Table of the values of ſingle lives, publiſhed in his *Treatiſe on Life Annuities*, and carried as far as the age of 79 to three places of decimals by Mr. *Dodſon* in his *Mathematical Repoſitory*, vol. ii. p. 169.

TABLE VI. Continued.

Age	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct.	5 per Ct.	6 per Ct.
34	16,057	15,039	14,126	13,304	12,562	11,278
35	15,864	14,871	13,979	13,175	12,449	11,189
36	15,666	14,699	13,829	13.044	12,333	11,098
37	15,465	14,524	13,676	12,909	12,214	11,003
38	15,260	14.345	13,519	12,771	12,091	10,907
39	15,053	14,163	13,359	12,630	11,966	10,807
40	14,842	13,978	13,196	12,485	11,837	10,704
41	14,626	13,789	13,028	12,337	11,705	10,599
42	14,407	13,596	12,858	12,185	11,570	10,490
43	14,185	13,399	12,683	12,029	11,431	10,378
44	13,958	13,199	12,504	11,870	11,288	10,263
45	13,728	12.993	12,322	11,707	11,142	10,144
46	13,493	12,784	12,135	11,540	10,992	10,021
47	13,254	12,571	11,944	11,368	10,837	9,895
48	13,012	12,354	11,748	11,192	10,679	9,765
49	12,764	12,131	11,548	11,012	10,515	9,630
50	12,511	11,904	11,344	10,827	10,348	9,492
51	12,255	11,673	11,135	10,638	10,176	9.349
52	11,994	11,437	10,921	10,443	9,999	9,201
53	11,729	11,195	10,702	10,243	9.817	9,049
54	11,457	10,950	10.478	10,039	9,630	8,891
55	11,183	10,698	10,248	9,829	9,437	8,729
56	10,902	10,443	10,014	9,614	9,239	8,561
57	10,616	10,181	9,773	9,393	9,036	8,387
58	10.325	9,913	9,527	9,166	8,826	8,208
59	10,029	9,640	9,275	8,933	8,611	8,023
60	9,727	9,361	9,017	8,694	8,389	7,831
61	9,419	9,076	8,753	8,449	8,161	7,633
62	9,107	8,786	8,482	8,197	7,926	7,428
63	8,787	8,488	8,205	7,938	7,684	7,216
64	8.462	8,185	7,921	7,672	7,435	6,997
65	8,132	7,875	7,631	7,399	7,179	6,770
66	7,794	7,558	7,333	7,119	6,915	6,535
67	7.450	7,234	7.027	6,831	6,643	6,292
68	7,099	6,902	6,714	6,534	6,362	6,040
69	6,743	6,565	6,394	6,230	6.073	5,779
70	6.378	6,219	6,065	5,918	5,775	5,508

TABLE VI. Continued.

Age.	3 per Ct.	3½ per Ct.	4 per Ct.	4½ per Ct	5 per Ct.	6 per Ct.
71	6,008	5,865	5,728	5,596	5,468	5,228
72	5,631	5,505	5,383	5,265	5,152	4,937
73	5,246	5,136	5,029	4,926	4,826	4,636
74	4,854	4,759	4,666	4,576	4,489	4,324
75	4,453	4,373	4,293	4,217	4,143	4,000
76	4,046	3,978	3,912	3,847	3,734	3,664
77	3,632	3,575	3,520	3,467	3,415	3,315
78	3,207	3,163	3,111	3.076	3,034	2,953
79	2,776	2,741	2,707	2,673	2,641	2,578
80	2,334	2,309	2,284	2,259	2,235	2,188
81	1,886	1,867	1,850	1,832	1,816	1,783
82	1,429	1,411	1,406	1,394	1,384	1,362
83	0,961	0,955	0,950	0,943	0,937	0,925
84	0,484	0,483	0,481	0,479	0,476	0,472
85	0,000	0,000	0,000	0,000	0,000	0,000

TABLE VII.

Shewing the Value of an Annuity on the joint continuance of two lives, according to Mr. *De Moivre's Hypotheſis*; and, therefore, nearly according to the probabilities of life at Breslaw, Norwich, and Northampton. See Eſſay II. and p. 2, 3, 226, 262.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
10	10	15.206	13.342	11.855
	15	14.878	13.093	11.661
	20	14.503	12.808	11.430
	25	14.074	12.480	11.182
	30	13.585	12.102	10.884
	35	13.025	11.665	10.537
	40	12.381	11.156	10.128
	45	11.644	10.564	9.646
	50	10.796	9.871	9.074
	55	9.822	9.059	8.391
	60	8.704	8.105	7.572
	65	7.417	6.980	6.585
	70	5.936	5.652	5.391
15	15	14.574	12.860	11.478
	20	14.225	12.593	11.266
	25	13.822	12.281	11.022
	30	13.359	11.921	10.736
	35	12.824	11.501	10.402
	40	12.207	11.013	10.008
	45	11.496	10.440	9.541
	50	10.675	9.767	8.985
	55	9.727	8.975	8.318
	60	8.632	8.041	7.515
	65	7.377	6.934	6.544
	70	5.932	5.623	5.364

TABLE VII. Continued.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
20	20	13.904	12.341	11.067
	25	13.531	12.051	10.840
	30	13.098	11.711	10.565
	35	12.594	11.314	10.278
	40	12.008	10.847	9.870
	45	11.325	10.297	9.420
	50	10.536	9.648	8.880
	55	9.617	8.879	8.233
	60	8.549	7.967	7.448
	65	7.308	6.882	6.495
	70	5.868	5.590	5.333
25	25	13.192	11.786	10.621
	30	12.794	11.468	10.367
	35	12.333	11.095	10.067
	40	11.776	10.655	9.708
	45	11.130	10.131	9.278
	50	10.374	9.509	8.761
	55	9.488	8.766	8.134
	60	8.452	7.880	7.371
	65	7.241	6.826	6.440
	70	5.826	5.551	5.294
30	30	12.434	11.182	10.133
	35	12.010	10.838	9.854
	40	11.502	10.428	9.514
	45	10.898	9.936	9.112
	50	10.183	9.345	8.620
	55	9.338	8.634	8.018
	60	8.338	7.779	7.280
	65	7.161	6.748	6.373
	70	5.777	5.505	5.254

TABLE VII. Continued.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
35	35	11.632	10.530	9.600
	40	11.175	10.157	9.291
	45	10.622	9.702	8.913
	50	9.955	9.149	8.450
	55	9.156	8.476	7.879
	60	8.202	7.658	7.172
	65	7.066	6.662	6.294
	70	5.718	5.450	5.203
40	40	10.777	9.826	9.014
	45	10.283	9.418	8.671
	50	9.677	8.911	8.244
	55	8.936	8.283	7.710
	60	8.038	7.510	7.039
	65	6.951	6.556	6.198
	70	5.646	5.383	5.141
45	45	9.863	9.063	8.370
	50	9.331	8.619	7.987
	55	8.662	8.044	7.500
	60	7.831	7.332	6.875
	65	6.807	6.425	6.080
	70	5.556	5.300	5.063
50	50	8.892	8.235	7.660
	55	8.312	7.738	7.230
	60	7.568	7.091	6.664
	65	6.623	6.258	5.926
	70	5.442	5.193	4.964
55	55	7.849	7.332	6.873
	60	7.220	6.781	6.386
	65	6.379	6.036	5.724
	70	5.291	5.053	4.833

TABLE VII. Continued.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
60	60	6.737	6.351	6.001
	65	6.043	5.730	5.444
	70	5.081	4.858	4.653
65	65	5.547	5.277	5.031
	70	4.773	4.571	4.385
70	70	4.270	4.104	3.952

TABLE VIII.

Shewing the Probability of the Duration of Life in LONDON, deduced by Mr. *Simpſon* from obſervations on the bills of mortality in LONDON for 10 years, from 1728 to 1737.

Ages	Perſons living.	Decr. of Life.	Ages.	Perſons living.	Decr. of Life.	Ages.	Perſons living.	Decr. of Life.
0	1000	320	27	321	6	54	135	6
1	680	133	28	315	7	55	129	6
2	547	51	29	308	7	56	123	6
3	496	27	30	301	7	57	117	5
4	469	17	31	294	7	58	112	5
5	452	12	32	287	7	59	107	5
6	440	10	33	280	7	60	102	5
7	430	8	34	273	7	61	97	5
8	422	7	35	266	7	62	92	5
9	415	5	36	259	7	63	87	5
10	410	5	37	252	7	64	82	5
11	405	5	38	245	8	65	77	5
12	400	5	39	237	8	66	72	5
13	395	5	40	229	7	67	67	5
14	390	5	41	222	8	68	62	4
15	385	5	42	214	8	69	58	4
16	380	5	43	206	7	70	54	4
17	375	5	44	199	7	71	50	4
18	370	5	45	192	7	72	46	4
19	365	5	46	185	7	73	42	3
20	360	5	47	178	7	74	39	3
21	355	5	48	171	6	75	36	3
22	350	5	49	165	6	76	33	3
23	345	6	50	159	6	77	30	3
24	339	6	51	153	6	78	27	2
25	333	6	52	147	6	79	25	
26	327	6	53	141	6			

TABLE IX.

Shewing the *Expectations* of Life in LONDON, according to the preceding Table. See Mr. *Simpson's Select Exercises*, p. 255.

Age.	Expectation.	Age.	Expectation.	Age.	Expectation.
1	27.0	28	24.6	55	14.2
2	32.0	29	24.1	56	13.8
3	34.0	30	23.6	57	13.4
4	35.6	31	23.1	58	13.1
5	36.0	32	22.7	59	12.7
6	36.0	33	22.3	60	12.4
7	35.8	34	21.9	61	12.0
8	35.6	35	21.5	62	11.6
9	35.2	36	21.1	63	11.2
10	34.8	37	20.7	64	10.8
11	34.3	38	20.3	65	10.5
12	33.7	39	19.9	66	10.1
13	33.1	40	19.6	67	9.8
14	32.5	41	19.2	68	9.4
15	31.9	42	18.8	69	9.1
16	31.3	43	18.5	70	8.8
17	30.7	44	18.1	71	8.4
18	30.1	45	17.8	72	8.1
19	29.5	46	17.4	73	7.8
20	28.9	47	17.0	74	7.5
21	28.3	48	16.7	75	7.2
22	27.7	49	16.3	76	6.8
23	27.2	50	16.0	77	6.4
24	26.6	51	15.6	78	6.0
25	26.1	52	15.2	79	5.5
26	25.6	53	14.9	80	5.0
27	25.1	54	14.5		

TABLE X.

Shewing the Value of an Annuity on *One* Life, according to the probabilities of life in LONDON. See Mr. *Simpſon's Select Exerciſes*, p. 260.

Age.	Yrs. purchaſe at 3 per Cent.	Yrs. purchaſe at 4 per Cent.	Yrs. purchaſe at 5 per Cent.	Age.	Yrs. purchaſe at 3 per Cent.	Yrs. purchaſe at 4 per Cent.	Yrs. purchaſe at 5 per Cent.	Age.	Yrs. purchaſe at 3 per Cent.	Yrs. purchaſe at 4 per Cent.	Yrs. purchaſe at 5 per Cent.
6	18.8	16.2	14.1	31	14.8	12.9	11.4	56	10.1	9.1	8.4
7	18.9	16.3	14.2	32	14.6	12.7	11.3	57	9.9	8.9	8.2
8	19.0	16.4	14.3	33	14.4	12.6	11.2	58	9.6	8.7	8.1
9	19.0	16.4	14.3	34	14.2	12.4	11.0	59	9.4	8.6	8.0
10	19.0	16.4	14.3	35	14.1	12.3	10.9	60	9.2	8.4	7.9
11	19.0	16.4	14.3	36	13.9	12.1	10.8	61	8.9	8.2	7.7
12	18.9	16.3	14.2	37	13.7	11.9	10.6	62	8.7	8.1	7.6
13	18.7	16.2	14.1	38	13.5	11.8	10.5	63	8.5	7.9	7.4
14	18.5	16.0	14.0	39	13.3	11.6	10.4	64	8.3	7.7	7.3
15	18.3	15.8	13.9	40	13.2	11.5	10.3	65	8.0	7.5	7.1
16	18.1	15.6	13.7	41	13.0	11.4	10.2	66	7.8	7.3	6.9
17	17.9	15.4	13.5	42	12.8	11.2	10.1	67	7.6	7.1	6.7
18	17.6	15.2	13.4	43	12.6	11.1	10.0	68	7.4	6.9	6.6
19	17.4	15.0	13.2	44	12.5	11.0	9.9	69	7.1	6.7	6.4
20	17.2	14.8	13.0	45	12.3	10.8	9.8	70	6.9	6.5	6.2
21	17.0	14.7	12.9	46	12.1	10.7	9.7	71	6.7	6.3	6.0
22	16.8	14.5	12.7	47	11.9	10.5	9.5	72	6.5	6.1	5.8
23	16.5	14.3	12.6	48	11.8	10.4	9.4	73	6.2	5.9	5.6
24	16.3	14.1	12.4	49	11.6	10.2	9.3	74	5.9	5.6	5.4
25	16.1	14.0	12.3	50	11.4	10.1	9.2	75	5.6	5.4	5.2
26	15.9	13.8	12.1	51	11.2	9.9	9.0				
27	15.6	13.6	12.0	52	11.0	9.8	8.9				
28	15.4	13.4	11.8	53	10.7	9.6	8.8				
29	15.2	13.2	11.7	54	10.5	9.4	8.6				
30	15.0	13.1	11.6	55	10.3	9.3	8.5				

TABLE XI.

Shewing the Value of an Annuity on the joint continuance of Two Lives, according to the probabilities of life in LONDON. See Mr. *Simpſon's Select Exerciſes*, p. 266.

Age of the youngeſt.	Age of the eldeſt.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
10	10	14.7	13.0	11.6
	15	14.3	12.7	11.3
	20	13.8	12.2	10.8
	25	13.1	11.6	10.2
	30	12.3	10.9	9.7
	35	11.5	10.2	9.1
	40	10.7	9.6	8.6
	45	10.0	9.0	8.1
	50	9.3	8.4	7.6
	55	8.6	7.8	7.1
	60	7.8	7.2	6.6
	65	6.9	6.5	6.1
	70	6.1	5.8	5.5
	75	5.3	5.1	4.9
15	15	13.9	12.3	11.0
	20	13.3	11.8	10.5
	25	12.6	11.2	10.1
	30	11.9	10.6	9.5
	35	11.2	10.0	9.0
	40	10.4	9.4	8.5
	45	9.6	8.8	8.0
	50	8.9	8.2	7.5
	55	8.2	7.6	7.0
	60	7.5	7.0	6.5
	65	6.8	6.4	6.0
	70	6.0	5.7	5.4
	75	5.2	5.0	4.8
20	20	12.8	11.3	10.1
	25	12.2	10.8	9.7
	30	11.6	10.3	9.2
	35	10.9	9.8	8.8
	40	10.2	9.2	8.4
	45	9.5	8.6	7.9
	50	8.8	8.0	7.4
	55	8.1	7.5	6.9
	60	7.4	6.9	6.4
	65	6.7	6.3	5.9
	70	6.0	5.7	5.4
	75	5.2	5.0	4.8
25	25	11.8	10.5	9.4
	30	11.3	10.1	9.0
	35	10.7	9.6	8.6
	40	10.0	9.1	8.2
	45	9.4	8.5	7.8
	50	8.7	7.9	7.3
	55	8.0	7.4	6.8
	60	7.3	6.8	6.3
	65	6.6	6.2	5.8
	70	5.9	5.6	5.3
	75	5.1	4.9	4.7
30	30	10.8	9.6	8.6
	35	10.3	9.2	8.3
	40	9.7	8.8	8.0

TABLE XI. Continued.

Age of the youngest.	Age of the eldest.	Value at 3 per Cent.	Value at 4 per Cent.	Value at 5 per Cent.
30	45	9.1	8.3	7.6
	50	8.5	7.8	7.2
	55	7.9	7.3	6.7
	60	7.2	6.7	6.2
	65	6.5	6.1	5.7
	70	5.8	5.5	5.2
	75	5.1	4.9	4.7
35	35	9.9	8.8	8.0
	40	9.4	8.5	7.7
	45	8.9	8.1	7.4
	50	8.3	7.6	7.0
	55	7.7	7.1	6.6
	60	7.1	6.5	6.1
	65	6.4	6.0	5.6
	70	5.7	5.4	5.1
	75	5.0	4.8	4.6
40	40	9.1	8.1	7.3
	45	8.7	7.8	7.1
	50	8.2	7.4	6.8
	55	7.6	6.9	6.4
	60	7.0	6.4	6.0
	65	6.4	5.9	5.5
	70	5.7	5.4	5.1
	75	5.0	4.8	4.6
45	45	8.3	7.4	6.7
	50	7.9	7.1	6.5
	55	7.4	6.7	6.2
	60	6.8	6.3	5.8
	65	6.3	5.8	5.4
	70	5.6	5.3	5.0
	75	4.9	4.7	4.5
50	50	7.6	6.8	6.2
	55	7.2	6.5	6.0
	60	6.7	6.1	5.7
	65	6.2	5.7	5.3
	70	5.5	5.2	4.9
	75	4.8	4.6	4.4
55	55	6.9	6.2	5.7
	60	6.5	5.9	5.5
	65	6.0	5.6	5.2
	70	5.4	5.1	4.8
	75	4.7	4.5	4.3
60	60	6.1	5.6	5.2
	65	5.7	5.3	4.9
	70	5.2	4.9	4.6
	75	4.6	4.4	4.2
65	65	5.4	5.0	4.7
	70	4.9	4.6	4.4
	75	4.4	4.2	4.0
70	70	4.6	4.4	4.2
	75	4.2	4.0	3.9
75	75	3.8	3.7	3.6

TABLE XII.

Shewing the Probabilities of Life in LONDON, on the ſuppoſition, that all who die in LONDON were born there. Formed from the Bills, for 10 years, from 1759 to 1768. See page 245.

Ages	Perſons living.	Decr. of Life.	Ages	Perſons living.	Decr. of Life.	Ages	Perſons living.	Decr. of Life.
0	1000	240	31	404	9	62	132	7
1	760	99	32	395	9	63	125	7
2	661	42	33	386	9	64	118	7
3	619	29	34	377	9	65	111	7
4	590	21	35	368	9	66	104	7
5	569	11	36	359	9	67	97	7
6	558	10	37	350	9	68	90	7
7	548	7	38	341	9	69	83	7
8	541	6	39	332	10	70	76	6
9	535	5	40	322	10	71	70	6
10	530	4	41	312	10	72	64	6
11	526	4	42	302	10	73	58	5
12	522	4	43	292	10	74	53	5
13	518	3	44	282	10	75	48	5
14	515	3	45	272	10	76	43	5
15	512	3	46	262	10	77	38	5
16	509	3	47	252	10	78	33	4
17	506	3	48	242	9	79	29	4
18	503	4	49	233	9	80	25	3
19	499	5	50	224	9	81	22	3
20	494	7	51	215	9	82	19	3
21	487	8	52	206	8	83	16	3
22	479	8	53	198	8	84	13	2
23	471	8	54	190	7	85	11	2
24	463	8	55	183	7	86	9	2
25	455	8	56	176	7	87	7	2
26	447	8	57	169	7	88	5	1
27	439	8	58	162	7	89	4	1
28	431	9	59	155	8	90	3	1
29	422	9	60	147	8			
30	413	9	61	139	7			

TABLE XIII.

Shewing the *true* Probabilities of Life in LONDON till the Age of 19. See page 249.

Age.	Perſons living.	Decrements of life.
0	750	240
1	510	99
2	411	42
3	369	29
4	340	21
5	319	11
6	308	10
7	298	7
8	291	6
9	285	5
10	280	4
11	276	4
12	272	4
13	268	3
14	265	3
15	262	3
16	259	3
17	256	3
18	253	4
19	249	
20	494	
21	487	
&c.	&c.	

The numbers in the ſecond column to be continued as in the laſt Table.

TABLE XIV.

Shewing the *true* Probabilities of Life in London for all Ages. Formed from the bills for 10 years, from 1759 to 1768. See page 251.

Ages.	Perſons living.	Decr. of Life.	Ages.	Perſons living.	Decr. of Life.	Ages.	Perſons living.	Decr. of Life.
0	1518	486	31	404	9	62	132	7
1	1032	200	32	395	9	63	125	7
2	832	85	33	386	9	64	118	7
3	747	59	34	377	9	65	111	7
4	688	42	35	368	9	66	104	7
5	646	23	36	359	9	67	97	7
6	623	20	37	350	9	68	90	7
7	603	14	38	341	9	69	83	7
8	589	12	39	332	10	70	76	6
9	577	10	40	322	10	71	70	6
10	567	9	41	312	10	72	64	6
11	558	9	42	302	10	73	58	5
12	549	8	43	292	10	74	53	5
13	541	7	44	282	10	75	48	5
14	534	6	45	272	10	76	43	5
15	528	6	46	262	10	77	38	5
16	522	7	47	252	10	78	33	4
17	515	7	48	242	9	79	29	4
18	508	7	49	233	9	80	25	3
19	501	7	50	224	9	81	22	3
20	494	7	51	215	9	82	19	3
21	487	8	52	206	8	83	16	3
22	479	8	53	198	8	84	13	2
23	471	8	54	190	7	85	11	2
24	463	8	55	183	7	86	9	2
25	455	8	56	176	7	87	7	2
26	447	8	57	169	7	88	5	1
27	439	8	58	162	7	89	4	1
28	431	9	59	155	8	90	3	1
29	422	9	60	147	8			
30	413	9	61	139	7			

OBSERVATIONS

ON

TABLES I. and II.

THESE Tables may be met with in moſt of the books that treat of compound intereſt and annuities; but there has been, in this work, ſo much occaſion for referring to them, that it was neceſſary to ſave the reader the trouble of turning to other books for them.

The 2d, 3d, 4th, &c. numbers in the *Second* Table, are only the *ſums* of the firſt 2, 3, 4, &c. numbers in the *Firſt* Table. This Table, therefore, is the foundation of the *Second*; and, indeed, of all the common tables of compound intereſt; and, with the help of it, almoſt all the queſtions in compound intereſt may be eaſily anſwered.

The following ſpecimen of this may, I think, be of conſiderable uſe.

QUESTION I. "To what *ſum* or *annuity* "will any given *ſum* or *annuity*, now to be "laid up for improvement, at a given rate "of compound intereſt, increaſe, in a given "number of years?"

ANSWER. Divide the given *ſum* or annuity by the value of 1*l.* payable at the end of the given number of years, and the *quotient* will be the anſwer.

Example. Let the given ſum be 50*l.* and the given time 18 years. The rate of intereſt 4 *per cent.*—The preſent value, at 4 *per cent.* of 1*l.* payable at the end of 18 years is, by Table I. .4936; and 50*l.* divided by this value, gives *l.*101.296, or 101*l.* 5*s.* the *ſum* to which 50*l.* will increaſe in 18 years. In like manner; 2*l. per annum*, the firſt payment of which is to be made a year hence, will increaſe (intereſt ſuppoſed the ſame) in 18 years, to an *annuity* of *l.*4.05: for 2*l.* the given annuity, divided by .4936, gives *l.*4.05, or 4*l.* 1*s.*

QUESTION II. "To what *ſum* will a "given *annuity* amount, in conſequence of "being forborn and improved, at a given "rate of compound intereſt, for a given "number of years?"

ANSWER. From the *increaſed* annuity, found by the laſt Queſtion, ſubtract the *given* annuity; and multiply the *remainder* by the PERPETUITY, and the *product* will be the anſwer.

Example. 2*l. per ann.* improved at 4 *per cent.* compound intereſt, will, by the laſt Queſtion, increaſe, in 18 years, to *l.*4.05 *per ann.* 2*l.* ſubtracted from 4.05, leaves 2.05, which, multiplied by 25, the *perpetuity*, gives *l.* 51.25, or 51*l.* 5*s*, the *amount* in 18 years. In the ſame manner it may be found, that 10*l. per ann.* (intereſt being the ſame) will amount, in 41 years, to 998*l.*

It ſhould be remembered, that the PERPETUITY is 33.33,—28.57,—25—20,—or 16.666, according as intereſt is reckoned at 3,—3½,—4,—5 or 6 *per cent:* And that the *annuity* meant in all theſe Queſtions is an annuity, the firſt payment of which is to be made at the end of a year.

QUESTION III. "In what number of "years will a given *ſum* or *annuity* increaſe "to another given *ſum* or *annuity*, in conſe"quence of being improved at a given rate "of intereſt?"

ANSWER. Divide the *original ſum* or *annuity* by the *increaſed ſum* or *annuity*; and look for the *quotient*, or the number neareſt to it, in Table I; and the number of years correſponding to it will be the anſwer.

Example. Let the *ſum* be 50*l.* The increaſed ſum *l.* 101.29. The rate of intereſt, 4 *per cent*. The former ſum divided by the latter gives .4936, which ſtands oppoſite in the Table to 18 years, or the time in which 50*l.* will gain the required increaſe.——In like manner, it may be found, that 18 years is the time in which 2*l. per ann.* will increaſe to *l.* 4.05 *per ann.*

Question IV. "In what time will any "given *annuity* amount to a *given ſum*, in "conſequence of being forborn and improved, at a given rate of compound intereſt?"

Answer. Divide the given *ſum* to which the annuity muſt amount by the perpetuity. Add the given annuity to the quotient; and by the quotient ſo increaſed, divide the given annuity; and this *ſecond quotient*, found in Table I. will ſhew the anſwer.

Example. A perſon owes 1000*l.* and reſolves to appropriate 10*l.* *per annum* of his income towards diſcharging it. In what time will ſuch an appropriation, in conſequence of being improved at 4 *per cent.* amount to a ſum equal to the debt?—— 1000*l.* divided by 25 gives 40*l.* 10*l.* added to 40*l.* makes 50*l*; and 10*l.* divided by 50*l.* gives .2000, which in the Table ſtands oppoſite to 41 years, the required time.

In the ſame manner it will appear, that the ſame annuity, if improved at 5 *per cent.* will amount to 1000*l.* in 37 years.

Question V. "In what time will a "*given principal* be annihilated, by taking "out of it, at the end of a year, a given ſum, "and after that, the ſame ſum annually, to"gether with its growing intereſts?"

ANSWER. In the ſame time plainly in which an equal annuity would amount to the *given principal*.

A perſon, therefore, poſſeſs'd of 1000*l.* capital, bearing intereſt at 4 *per cent.* would, by Queſtion IV. reduce it to nothing in 41 years, by taking out of it 10*l.* the firſt year, and as much more every following year, as would be neceſſary, together with the intereſt of the remaining capital, to make his annual income conſtantly 50*l.*

Remark. The ſum to which a *given* annuity will amount in a given time, is the ſame with the value of an annuity for the given time, equal to the *given* annuity increaſed by the yearly intereſt of the amount. That is, 1000*l.* is the value of 50*l.* *per ann.* for 41 years, at 4 *per cent:* And the ſame ſum is likewiſe the value of 60*l.* *per annum*, for 37 years, at 5 *per cent.* The reaſon is plain: 1000*l.* it has appeared, would, in conſequence of being put out to theſe different rates of intereſt, be juſt ſufficient to pay the annuities.

I have been the more explicit in theſe rules, becauſe they point out a very eaſy method of deducing and examining all I have ſaid, in different parts of this work, and particularly in Chap. III. concerning the increaſe of money at intereſt.——I will juſt mention one inſtance.

400,000*l.* *per annum*, applied in the manner ſuppoſed in Queſtions IV and V. would annihilate 55 millions, bearing intereſt at 5 *per cent.* in 42 years.

In 1716, when the *ſinking fund* was eſtabliſhed, the public debts were near this ſum, and bore 5 *per cent.* intereſt. This fund then, had but 400,000*l.* of it been inviolably applied to the annihilation of the public debts, would, in 1758, have diſcharged all the debts contracted *before* 1716.——And it may be further found very eaſily, by the anſwer to Queſtion IV. that had it been ſuffered to go on in its operation, and been applied, *ſince* 1758, to the redemption of only 3 *per cents* at *par*, it would by this time have diſcharged 104 millions; and ſeven years hence, 140 millions.——The aſſertion, therefore, in page 164, is ſtrictly true. But the following proof of that aſſertion will, perhaps, be more clear and ſtriking.

Suppoſe an annuity of 400,000*l*, beginning in 1716, to have been applied till 1730, to the annihilation of debts bearing intereſt at 5 *per cent*; from 1730 to 1748, to the annihilation of debts bearing intereſt at 4 *per cent*. and from 1748 to 1771, to the annihilation of debts bearing intereſt at 3 *per cent*. In the firſt of theſe periods the annuity would have increaſed to 800,000*l.*; in the *ſecond*, to 1,600,000*l.*; in the *laſt*, to 3,200,000*l.*——In the preſent year, therefore, the nation might have been eaſed of above *three millions per annum* in taxes. And, at the ſame time, (ſuppoſing all the ſame meaſures taken in other reſpects) it would have enjoyed that very *ſinking fund* it now has; and no detriment could have ariſen to the public, from any applications of it to current expences.

As I am now again on the national debt, I will beg leave to add the following proof of another obſervation on this ſubject, in page 140.

The diſburſements, on account of any loans, will be the ſame, whatever different intereſts they bear, ſuppoſing a proviſion made for diſcharging the principal, by applying to that purpoſe ſurpluſſes bearing to one another the ſame proportions with thoſe intereſts.

For Example. Let a million be borrowed at 3 *per cent.* and let a fund be charged with it which brings in 6*s. per cent. per annum* more than the intereſt; or 33,000*l.* inſtead of 30,000*l. per annum.* This ſurplus, applied in the manner I have explained, will annihilate the *principal* in 81 years, as may be gathered from Queſtion V. At the end of this time, the diſburſements on account of the loan will be 33,000*l.* multiplied by 81; that is, 2,673,000*l.*

Let us again ſuppoſe a million borrowed at 6 *per cent.* and let a fund be charged with it producing a ſurplus of 12*s. per cent. per annum.*

annum. Such a fund, beſides paying the intereſt, will diſcharge the principal in 41 years; and the diſburſements on account of the loan, will be 66,000*l.* multiplied by 41; that is, 2,706,000*l.* or very nearly the ſame with the diſburſements on account of an equal loan at 3 *per cent*.

It appears, therefore, agreeably to the obſervation to which I have referred, that were the public, in raiſing money, to adopt the plan I have propoſed, it would be of little conſequence what intereſt was given for money. The practicability of ſuch a plan is ſelf-evident, for it cannot be leſs eaſy to apply the *intereſt* of a ſum, to the payment of a debt, than the ſum itſelf: and this plan requires no more.—One particular advantage attending it, already hinted, I will beg leave here to repeat. By keeping the *ſtocks* ſteadily at or near *par*, that fluctuation in them would be in a great meaſure prevented, which now produces ſo many evils; and which, with the aid of annual lotteries, will, I fear, in time, ruin all honeſt induſtry, and turn us into a nation of ſharpers and gamblers.

Directions for finding the Values of Two Joint Lives of given Ages, by Table VII.

If both the ages are given in the Table, the value wanted will be found immediately by inſpection.

If the ages are not given in the Table, it will be beſt to proceed in the following manner.

Suppoſe the rate of intereſt 4 *per cent.* and the propoſed ages 40 and 66.——It will appear, from inſpecting the Table, that the value ſought would be 6.556, were the age of the elder life 65; and 5.383, were it 70. Since, therefore, it is 66, the value muſt be the *firſt* of four arithmetical means between 6.556 and 5.383, or 6.322.——For the ſame reaſon, had the ages of the elder been 68, the value would have been the 3d arithmetical mean between 6.556 and 5.383, or 5.854.——In like manner, were the propoſed ages 43 and 65, the value would be the 3d arithmetical mean between 6.556 (the value of two joint lives whoſe ages are 40 and 65) and 6.425, (the value of two joint lives whoſe ages are 45 and 65) or 6.478.

Again, let the ages be 43 and 66. That is, let it be ſuppoſed, that neither of the propoſed ages is given in the Table.

The values correſponding to the ages $\left\{\begin{matrix}40\\45\end{matrix}\right\}$ and $\left\{\begin{matrix}66\\66\end{matrix}\right\}$, are $\left\{\begin{matrix}6.322\\6.200\end{matrix}\right\}$.

The value, therefore, correſponding to the ages 43 and 66, muſt be the 3d mean between 6.322 and 6.200, or 6.250.——*N. B.* The 1ſt, 2d, 3d, and 4th of four arithmetical means between two numbers are found by ſubtracting $\frac{1}{5}$, $\frac{2}{5}$, $\frac{3}{5}$, and $\frac{4}{5}$ of the *difference* between the two numbers, from the *greateſt* of them.

Thus. The difference between 6.556, and 5.383, is 1.173. One fifth of this difference is .234; which, ſubtracted from 6.556, leaves 6.322; the firſt of 4 means between 6.556 and 5.383.——In like manner; the difference between 6.322 and 6.200 is .122. *One* fifth of this difference is .024; and, therefore, three-fifths of this difference is .072, which, ſubtracted from 6.322, leaves 6.250, the *third* arithmetical mean between 6.322 and 6.200.

In order to avoid trouble, if the ages are nearly equal, a year or two may be added to the leaſt; and as much ſubtracted from the greateſt; and the value taken by inſpection. But if one of them much exceeds the other, it will in general be ſufficient to take the neareſt number in the Table for the leſſer.

The mean between the values at 3 *per cent.* and 4 *per cent.* may be taken for the value at $3\frac{1}{2}$ *per cent.* without any error of conſequence. And the like may be ſaid of the values at $4\frac{1}{2}$ *per cent.*

The values of the *longeſt* of two lives is found by ſubtracting the value of the *joint* lives from the *ſum* of the values of the two *ſingle* lives.——Thus, the values of two ſingle lives, whoſe ages are 25 and 30, are by Table VI. (intereſt reckoned at 4 *per cent.*) 15.31 and 14.68. The ſum of theſe two values is 29.99; the value of the joint lives is (by Table VII) 11.46; and this value, ſubtracted from 29.99, gives 18.53, or the value of an annuity on the longeſt of the two lives.

The value of *two* joint lives being given, the value of *three* joint lives may be found by the following rule, taken from Mr. *Simpſon's Select Exerciſes*, page 279.

Let A be the youngeſt, and C the oldeſt of the three propoſed lives. Take the value of the two joint lives B and C, and find the age of a *ſingle* life D of the ſame value. Then find the value of the *joint* lives A and D, which will be the anſwer.

Example. Let the three given ages be 25, 30, and 40, and let the rate of intereſt be 4 *per cent.* Then the value of the two oldeſt joint lives B and C, will (by Tab. VII.) be 10.428, anſwering, in Tab. VI. to a ſingle life D of 54 years of age. And the value of the joint lives A and D, which is 8.917 year's purchaſe, will be the value ſought.

FINIS.

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